Stiffness Control of Balance in Quiet Standing

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Winter, David A., Aftab E. Patla, Francois Prince, Milad Ishac, and Krystyna Giele-Perczak. Stiffness control of balance in quiet standing. J. Neurophysiol. 80: 1211–1221, 1998. Our goal was to provide some insights into how the CNS controls and maintains an upright standing posture, which is an integral part of activities of daily living. Although researchers have used simple performance measures of maintenance of this posture quite effectively in clinical decision making, the mechanisms and control principles involved have not been clear. We propose a relatively simple control scheme for regulation of upright posture that provides almost instantaneous corrective response and reduces the operating demands on the CNS. The analytic model is derived and experimentally validated. A stiffness model was developed for quiet standing. The model assumes that muscles act as springs to cause the center-of-pressure (COP) to move in phase with the center-of-mass (COM) as the body sways about some desired position. In the sagittal plane this stiffness control exists at the ankle plantarflexors, in the frontal plane by the hip abductors/adductors. On the basis of observations that the COP-COM error signal continuously oscillates, it is evident that the inverted pendulum model is severely underdamped, approaching the undamped condition. The spectrum of this error signal is seen to match that of a tuned mass, spring, damper system, and a curve fit of this “tuned circuit” yields $w_n$, the undamped natural frequency of the system. The effective stiffness of the system, $K_e$, is then estimated from $K_e = I w_n^2$, and the damping $B$ is estimated from $B = BW \times I$, where $BW$ is the bandwidth of the tuned response (in rad/s), and $I$ is the moment of inertia of the body about the ankle joint. Ten adult subjects were assessed while standing quietly at three stance widths: 50% hip-to-hip distance, 100 and 150%. Subjects stood for 2 min in each position with eyes open; the 100% stance width was repeated with eyes closed. In all trials and in both planes, the COP oscillated virtually in phase (within 6 ms) with COM, which was predicted by a simple 0th order spring model. Sway amplitude decreased as stance width increased, and $K_e$ increased with stance width. A stiffness model would predict sway to vary as $K_e^{0.55}$. The experimental results were close to this prediction: sway was proportional to $K_e^{0.55}$. Reactive control of balance was not evident for several reasons. The visual system does not appear to contribute because no significant difference between eyes open and eyes closed results was found at 100% stance width. Vestibular (otolith) and joint proprioceptive reactive control were discounted because the necessary head accelerations, joint displacements, and velocities were well below reported thresholds. Besides, any reactive control would predict that COP would considerably lag (150–250 ms) behind the COM. Because the average COP was only 4 ms delayed behind the COM, reactive control was not evident; this small delay was accounted for by the damping in the tuned mechanical system.

INTRODUCTION

The ability to stand upright on two feet is important in and of itself or as a precursor to initiation of other activities of daily living. This ability acquired early in life is performed automatically and in some sense seems hardly worth a second glance from scientists interested in the study of balance and postural control. And yet this simple activity has been studied extensively and has yielded a rich source of insights into the postural control system. Because of its simplicity and relative ease of evaluating performance, it has a long history of use in clinical settings (Diener et al. 1984a). Although “time to maintain a given posture” is a useful clinical measure, most other studies of upright posture use a measure of “body sway” to characterize the performance. Implicit underlying assumption in the measure of body sway is that body center-of-mass (COM) is what is regulated in the gravitational environment. Body sway is a kinematic term and is often estimated from center-of-pressure (COP) measures derived from force plate data, and even erroneously assumed to be synonymous to the COP measure.

Estimation of a COP of multisegment human body requires kinematic measurement of all body segment displacements and an anthropometric model of the body (Winter 1990). Accurate estimation of small movements of body COM requires very precise measures of proximal and distal displacements of all individual body segments; once this is available, an anthropometric model to derive body COM can be easily implemented. Because there is no single sensory receptor or modality that can directly measure body COM, the complex pattern of sensory input has to be processed. Many studies have shown that when various sensory systems are systematically manipulated, body sway is affected. For example, elimination of pressor receptors under the feet through ischemic blocking increases body sway (Diener et al. 1984b). Absence of visual input also has been shown to result in an increase in body sway. Similarly galvanic stimulation of the vestibular apparatus, and ankle muscle vibration result in increased body sway that is directionally specific (Fitzpatrick et al. 1994). Although there is a specific physiological mapping between galvanic stimulation or muscle vibration and direction of body sway, similar mapping between absence of visual sensory input or pressure receptor output and body sway is not clear. The latter and not the former is the norm under normal conditions, assuming that the relatively small body movements during quiet standing are sufficient to stimulate the various sensory receptors. The complex pattern of sensory input that may be delivered to the CNS during maintenance of quiet posture may in theory be able to provide an estimate of body COM.

This paper presents a relatively simple control scheme for
the control of upright posture, such that in this posture, the body behaves like an inverted pendulum. Body COM is regulated through movement of the COP under the feet. In such a model the difference between body COM and COP will be proportional to the acceleration of body COM. The COP is controlled by ankle plantarflexors/dorsiflexors torque in the sagittal plane and hip abductor/adductor torque in the frontal plane. We propose this restoration torque is set by the joint stiffness. Thus CNS setting of joint stiffness through appropriate muscle tone is a simple way of controlling body COM during quiet standing.

**THEORETICAL MODEL**

**Inverted pendulum model**

The inverted pendulum model relates the controlled variable (COM) with the controlling variable (COP). Such a model provides an analytic relationship between these two commonly measured variables and the horizontal acceleration of the COM. This relationship derived in Appendix A shows that the COP-COM is indeed proportional to the horizontal acceleration of the COM in both the sagittal (anterior/posterior direction, A/P) and frontal (medial/lateral direction, M/L) planes. The two equations that capture this relationship are

\[ (p_x - x) = (-I_p/Wh)x\]

(1)

where \( p_x \) is the center of pressure position with respect to the ankle joint in the A/P direction, \( x \) is the COM position with respect to the ankle joint in the A/P direction, \( I_p \) is the COM horizontal acceleration, \( I_{ap} \) is the inertia of the body about the ankle joint in the sagittal plane, \( W \) is the weight of the body (minus the weight of the feet), and \( h \) is the COM height above the ankle joint and

\[ (p_z - z) = (-I_p/Wh)z\]

(2)

where \( z \) refers to displacements in the M/L direction and \( I_p \) is the inertia of the body about the ankle joint in the frontal plane.

Thus the inverted pendulum model (Eqs. 1 and 2) in both A/P and M/L planes states that the horizontal acceleration of the pendulum is proportional to the difference between the COP and COM. An identical relationship was developed by Brenière (1996) using similar simplifications but assuming that COP and COM were periodic functions in phase with each other. If the COP is ahead of the COM, the COM is being accelerated backward and vice versa if the COP is behind the COM. Similarly, if the COP is to the right of the COM, the mass is being accelerated to the left, and to the right if the COP is to the left of the COM.

The first aim of this paper is to validate the inverted pendulum model (Eqs. 1 and 2) in both A/P and M/L planes. This validation will be achieved by demonstrating a high correlation between the COP-COM error signal and the respective horizontal accelerations of the COM in each plane.

**Stiffness control model the inverted pendulum**

As has been reported in the few papers that have modeled the total body COM with reasonable accuracy (Hasan et al. 1996; Jian et al. 1993; Winter 1990), the COP tracks the COM oscillating either side of it to maintain it in some central position between the two feet. In the sagittal plane, the ankle plantarflexor/dorsiflexor moments control COM during quiet standing. But because the ankle moment, \( M_F = p_x \), and \( R \), the vertical reaction force at the ankle, is a constant, we consider \( p_x \) to be the controlling motor variable that is readily measured from force platforms. In the frontal plane the net moment acting on the closed loop is \( M_L = R p_z \).

Again, because \( R \) is a constant, the M/L COP, \( p_z \), is the readily measured controlling variable that reflects the load/unloading of the limbs by the hip abductors/adductors (Winter et al. 1996). Therefore, in both cases, the COM is the controlled variable, whereas the COP is the controlling variable.

We hypothesize a simple stiffness model that produces the appropriate joint moment and suggest that the CNS sets the muscle tone at specific balance control sites such that the stiffness constant is sufficient to control the large inertial load against the gravitational forces that attempt to topple the pendulum system. Consider a simplified stiffness control model of the inverted pendulum where a rotational spring creates a moment at the base of the pendulum (the ankle joint).

In static condition, the moment due to the spring, \( K\theta \), balances the moment about the ankle joint, \( Wh \sin \theta \), where \( K \) is the rotational spring stiffness in Nm/rad and \( \theta \) is the angle of the pendulum from vertical. Thus

\[ K\theta = Wh\theta \]

For small angles of sway

\[ \theta \approx x/h \]

Therefore, the effective stiffness of the inverted pendulum defined as \( K = Wh \) (the gravitational spring, \( Wh \), acts to reduce the stiffness).

It is clear that this simplified stiffness model will oscillate at \( \omega_n \). Any small damping present will result in the COM oscillations decaying to zero. During quiet standing the COP and COM excursions do not oscillate at a single frequency (see Fig. 1, A and B), and these oscillations continue. This implies that energy is continuously being generated into this mass, spring, and damper system creating a tuned mechanical circuit. Because stiffness \( K \) determines the acceleration of the COM, and from the inverted pendulum model COP-COM is proportional to the acceleration of the COM, we can estimate \( K \) by analyzing the amplitude spectrum of the COP-COM signal. The amplitude spectrum of the COP-COM signal calculated using a fast Fourier transform and converted to a log scale, is shown in Fig. 2. This spectrum represents the response of a tuned mechanical circuit. The equation of the amplitude spectrum of a tuned mechanical system is described by

\[ A(\omega) = \frac{C}{\sqrt{1 + \left(\frac{I\omega}{B} - \frac{K}{\omega B}\right)^2}} \]

(4)

where \( I \), \( K \), and \( B \) are the inertial, spring, and damping constants, and \( C \) is a constant. \( I \) is determined by anthropometric measures (Winter 1990). This response reaches a maximum when \((I\omega/B - K/\omega B) = 0 \) or when \( \omega_n = \sqrt{K/I} \). This equation is the mechanical analogue of a standard electrical tuned circuit (see Ogata 1992).

A curve fit of this tuned mechanical system response yields \( \omega_n \), the undamped resonant frequency of the system; the optimization
A: typical 40-s record from a subject standing quietly. Center-of-pressure (COP) and center-of-mass (COM) in anterior/posterior direction (A/P) show the COP to “track” the COM almost in phase and to be oscillating either side of the COM. 

B: COP-COM signal for the same 40-s record showing this “error” signal to have a band of frequencies centered on $f_n$, the undamped natural frequency of the inverted pendulum.

The optimization program to achieve this fit varies $C$, $K_e$, and $B$ with $I$ set to the subject’s $I$. $K_e$ and $B$ can be determined two ways. The optimization program can yield $K_e$ and $B$. Alternately $K_e$ can be calculated from Eq. 3 and $B = BW \times I$, where $BW$ is the bandwidth of the tuned mechanical system. Thus we have an analytic way of estimating the stiffness and damping of the inverted pendulum, which controls upright balance.

One prediction from the simplified (undamped) stiffness control model of the inverted pendulum is that magnitude of sway, $x(t)$, is

$$x(t) = \sqrt{\frac{I \omega_n^2 + V_0^2}{\omega_n^2}} \sin(\omega t + \phi)$$

where $V_0$ is the horizontal velocity of the COM when it is at “top dead center” and $x_0$ is the horizontal displacement of the COM at $t = 0$.

If we start the pendulum oscillating at $x_0 = 0$ at $t = 0$ the amplitude of the oscillation is

$$X = \sqrt{\frac{V_0^2}{\omega_n^2}} = \frac{V_0}{\sqrt{K_e/I_{cm}}} = \frac{V_0 \sqrt{I}}{\sqrt{K_e}}$$

Thus the displacement of COM is proportional to $K_e^{-0.5}$. Figure 3 shows this relationship for three different values of $V_0$. Note that, although COM displacement is affected by the magnitude of initial velocity, the curve relating COM displacement and $K_e$ have the same shape.

Three predictions from the proposed model will be tested to validate the stiffness control model. First, we will show that the
COP and COM profiles will be in phase because of the dominant influence of stiffness on the behavior (as would be predicted in a 0th order system). In the A/P plane, stiffness is set by the ankle plantar/dorsiflexors, whereas, in the M/L plane, hip abductors/adductors set the spring stiffness. Second, we will vary the stiffness by changing the posture in the mediolateral plane and show that curve fit between magnitude of COM displacement and stiffness is as predicted by the model. Third, we will show that the stiffness set by the CNS fluctuates about an average value giving rise to the measured complex oscillation of the COP and COM signal as predicted by the response of a tuned mechanical system.

Experimental protocol

A 14-segment model was developed to estimate the total body COM. It consisted of legs (2), thighs (2), lower arms (2), upper arms (2), pelvis and trunk (4). Figure 4 gives the location of the 21 infrared emitting diodes (IREDs) that were tracked by a 3D OPTOTRAK imaging system. The definition of each segment and the mass fraction of each segment is presented in Table 1. Ten young adults (average age, 26 yr; body mass, 68.8 ± 9.1 kg, mean ± SD) with no known balance or gait pathology were analyzed. Informed consent was obtained from each subject. Subjects were instructed to stand quietly on two Advanced Mechanical Technology force platforms with the feet in the side-by-side position at three different widths. The purpose of three different widths is to have a variable base width and a variable sway amplitude to test the prediction that sway will be proportional to \( K_0^{0.5} \). The 100% width had the ankle-to-ankle distance equal to the distance between the hip joints, where the hip joint distance was estimated to be equal to the distance between the right and left anterior superior iliac spine. The 50% width had the ankle-to-ankle distance = 0.5 of the hip joint distance and the 150% width had the ankles spaced at 1.5 times the hip joint distance. Typical values for the stance width in this study were 42, 28, and 14 cm. Subjects were instructed to stand quietly in each position for 2 min with eyes open. The 100% width position was repeated with eyes closed. Two minutes was chosen because shorter length records failed to capture the very low frequencies present in the COM and COP trajectories (Powell and Dzendolet 1984). COP and COM was measured in a rigid manikin to estimate the net measurement noise in the force platforms and OPTOTRAK systems.

The COM is a weighted average of the COM’s of each of the 14 segments, in the x direction

\[
\text{COM}(x) = \frac{1}{M} \sum_{i=1}^{14} \text{COM}_i(x) \cdot m_i
\]

where \( M \) is total body mass, \( m_i \) is mass of \( i \)th segment, and \( \text{COM}_i(x) \) is \( x \) coordinate of \( i \)th segment. The OPTOTRAK sampled
the light-emitting diode (LED) data at 20 Hz in all three dimensions. x was positive in the forward direction, y was positive vertically, and z was positive to the right. The location of the estimated COM of each segment is shown in Table 1 along with the mass fraction of each segment, $m_i/M$. Thus the COM is estimated in three-dimensional space every 50 ms. The COP in the A/P and M/L directions were calculated from the force platform data using the following equation (Winter et al. 1993, 1996):

$$\text{COP}(t) = \frac{R_{\text{L}}(t)}{R_{\text{L}}(t) + R_{\text{R}}(t)} + \frac{R_{\text{R}}(t)}{R_{\text{L}}(t) + R_{\text{R}}(t)}$$

where $\text{COP}_{\text{L}}(t)$ and $\text{COP}_{\text{R}}(t)$ are the COPs under the left and right feet, respectively, and $R_{\text{L}}(t)$ and $R_{\text{R}}(t)$ are the ground reaction forces under the left and right feet, respectively.

We consider our COP measures to be quite accurate because they are direct measures from calibrated force platforms. However, our COM estimates of each segment may have a bias error. For example, the true segment COM for the thoracic region may be 1 cm anterior of where we estimate. However, during quiet standing or voluntary swaying, this bias will remain essentially constant over the data collection period. The same applies to all 14 segments. Thus there may be a constant (bias) error in our COM. We know from our theoretical model (Eqs. 1 and 2) that the COP-COM = ($-I/Wh$)$\bar{x}$. Thus we can remove the net bias error from COM by letting the average position of COM be equal to the average position of COP over the 2-min period. The validity of such an assumption was demonstrated by the fact that after the bias removal the COP was seen oscillating either side of the COM for the entire 2 min and that there was a high correlation between COP-COM and the horizontal acceleration in either A/P or M/L directions.

A curve fit of Eq. 4 to the amplitude spectrum of the (COP-COM) yielded $f_e$ from which $K_e$ was calculated. Note that $f_e$ (Hz) is related to $\omega_e$ (rad/s) in Eq. 3 by $\omega_e = 2\pi f_e$. The details of the curve fitting procedure is the subject of a separate technical note (K. Gielo-Perczak, D. A. Winter, and A. E. Patla, unpublished observations).

The time shift difference between COP ($p_x$) and COM ($x$) was determined by the peak of the cross-correlation of the $p_x(t)$ and $x(t)$ signals

$$R_{xy}(\tau) = \frac{1}{T}\int_{-T/2}^{T/2} p_x(t)x(t+\tau)dt$$

where $T$ is the duration of the signal and $R_{xy}(\tau)$ is the cross-correlation function value for a time shift difference of $\tau$. The time shift is recorded when $R_{xy}(\tau)$ is maximum and is negative when COP lags COM. The signals $p_x(t)$ and $x(t)$ were interpolated to 10 ms to increase the precision of time difference, $\tau$.

**RESULTS**

Figure 1A, which has already been reported, is a representative COP and COM plots for 40 s of quiet standing in the A/P direction showing how closely COP and COM are in phase and how COP oscillates either side of COM. This subject was standing in the 50% stance width position with eyes open. Figure 1B is the (COP-COM) signal for this same subject and shows the oscillating nature of this "error" signal. Figure 2, as previously reported, is the amplitude spectrum of this (COP-COM) signal showing the tuned mechanical curve fit that was used to estimate $f_e$, $K_e$, $BW$, and $B$ for the second-order inverted pendulum. Figure 5, A and B, are representative plots of (COP-COM) versus acceleration of COM in the A/P and M/L directions, respectively. It is noted that these two signals are 180° out of phase, which is predicted by the negative sign in Eqs. 1 and 2 for the inverted pendulum model.

**TABLE 1. Segment definitions**

<table>
<thead>
<tr>
<th>Segment</th>
<th>Mass Fraction</th>
<th>Marker Definition of COM</th>
</tr>
</thead>
<tbody>
<tr>
<td>Head</td>
<td>0.081</td>
<td>($13 + 14i)/2</td>
</tr>
<tr>
<td>Trunk 4</td>
<td>0.136</td>
<td>($9 + 12 + 21i)/3</td>
</tr>
<tr>
<td>Trunk 3</td>
<td>0.078</td>
<td>($19 + 20i)/2 + 21i)/2</td>
</tr>
<tr>
<td>Trunk 2</td>
<td>0.065</td>
<td>($17 + 18 + 19 + 20i)/4</td>
</tr>
<tr>
<td>Trunk 1</td>
<td>0.078</td>
<td>($17 + 18 + 15 + 16i)/4</td>
</tr>
<tr>
<td>Pelvis</td>
<td>0.142</td>
<td>($15 + 16i)/2</td>
</tr>
<tr>
<td>Thighs</td>
<td>0.100 (2)</td>
<td>0.433 × 3 + 0.567 × 5</td>
</tr>
<tr>
<td>Legs and feet</td>
<td>0.060 (2)</td>
<td>and 0.433 × 4 + 0.567 × 6</td>
</tr>
<tr>
<td>Upper arms</td>
<td>0.028 (2)</td>
<td>0.606 × 1 + 0.394 × 3</td>
</tr>
<tr>
<td>Fore arms</td>
<td>0.022 (2)</td>
<td>and 0.606 × 2 + 0.394 × 4</td>
</tr>
<tr>
<td>Total</td>
<td>1.000</td>
<td>0.682 × 7 + 0.318 × 8</td>
</tr>
<tr>
<td></td>
<td></td>
<td>and 0.682 × 10 + 0.318 × 11</td>
</tr>
</tbody>
</table>

COM, center-of-mass.
The COP tracks the COM and oscillates either side of it to stabilize it around some central position. No significant A/P differences were found between any of the stance width positions because the base of support in the A/P direction remained constant. However, in the M/L direction both the sway (COM) and COP amplitudes decrease significantly as stance width increases. The 150% stance width COM (0.094 cm) was significantly smaller ($P < 0.03$) than the 100% stance width COM (0.166 cm), which, in turn was significantly smaller ($P < 0.01$) than during the 50% stance width (0.277 cm). In all stance conditions the error signal (COP-COM) in the M/L direction was ~0.055 cm and ~0.08 cm in the A/P direction. The $f_n$ were not significantly different in the A/P direction for all conditions. However, in the M/L direction $f_n$ increased significantly with stance width. In the 100% stance width $f_n$ (0.680 ± 0.141 Hz) was significantly higher ($P < 0.0005$) than $f_n$ for the 50% stance width (0.496 ± 0.091 Hz). In turn, the 150% stance width $f_n$ (1.12 ± 0.32 Hz) was significantly higher ($P < 0.001$) than in the 100% stance width. In all COP and COM measures and all estimates of $f_n$, $K_e$, and $B$, there were no significant differences between the eyes closed trials at 100% stance width and the eyes open trials at the same width. The damping constant $B$ was very constant for all conditions in the A/P direction. However, there was one significant difference ($P < 0.05$) between the 150% stance (332 ± 128) and 100% stance (205 ± 101). The increased damping between these two conditions was in the same direction as the differences in $K_e$, suggesting that the damping coefficient ($B/(2IK_e)^{1/2}$) is being maintained constant. The time shift between the COP and COM was very small; over all A/P and M/L trials the COP lagged the COM by 4 ms (negative time shift in Table 3 means COP is delayed behind COM).

**DISCUSSION**

**Validity of the inverted pendulum model**

In both A/P and M/L directions, it is evident from that the COP tracks the COM and oscillates either side of it to keep the COM within a desired position between the two feet. Because the COP oscillates either side of the COM,

**TABLE 2. Correlation coefficient between COP-COM and COM acceleration**

<table>
<thead>
<tr>
<th>Direction</th>
<th>A/P</th>
<th>M/L</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stance width</td>
<td>50% EO</td>
<td>100% EO</td>
</tr>
<tr>
<td>Correlation</td>
<td>$-0.902 \pm 0.048$</td>
<td>$-0.914 \pm 0.039$</td>
</tr>
</tbody>
</table>

Values in Correlation are means ± SD. COP, center-of-pressure; COM, center-of-mass; A/P, anterior/posterior; M/L, medial/lateral.
the COP displacement is always slightly larger than the COM. The COP-COM represents the error signal as COP tracks the COM. From Table 3, it is evident that this correction signal is ~0.8 mm in the A/P direction and ~0.5 mm in the M/L direction. The inverted pendulum equations (Eqs. 1 and 2) show that this error signal is proportional to linear horizontal acceleration of the COM and therefore may represent the error signal driving reactive feedback, or it may represent a simpler error in the difference between the stiffness torque and the gravitational torque. This is explored in detail in the next section.

The correlation coefficient between the COP-COM and the horizontal accelerations (Table 2) are the primary measures of the validity of the inverted pendulum model. These correlations were quite high in the A/P direction and slightly lower in the M/L direction. These smaller correlations were attributed to the smaller magnitude signals being correlated in the M/L direction than in the A/P direction. At very low amplitudes the precision of the COM estimates (Eq. 6) decreases, and also we are now approaching the precision of the OPTOTRAK and force platform systems. The noise level of the total OPTOTRAK/force platform systems was estimated by calculating COP-COM from a rigid manikin standing on the force platforms and instrumented with LEDs the same as the subjects. The noise was 0.17 mm in the A/P direction and 0.27 mm in the M/L direction. The COP-COM signal in the M/L direction ranged from 0.46 to 0.55 mm, which is only about two times the noise level, whereas in the A/P direction COP-COM ranged from 0.71 to 1.01 mm, which was about five times the A/P noise. In experiments when subjects made large voluntary sways in the A/P or M/L directions, the correlations between the COP-COM and the horizontal accelerations consistently range between ~0.96 and ~0.99. An inverted pendulum model has also been used to explain the A/P and M/L accelerations of the COM during initiation and termination of gait (Jian et al. 1993). During the period of time between initial acceleration forward and laterally toward the stance limb until toe off, the correlation between COP-COM and the COM acceleration averaged ~0.93. Similarly, during the final stage of gait termination when both feet are on the ground, the correlations also averaged ~0.93.

**Validity of the stiffness control model of the inverted pendulum**

The first prediction states that COP should oscillate effectively in phase with COM. A visual inspection of Fig. 1A demonstrates this in-phase relationship for a typical subject in the A/P direction. The COP moves and tracks the COM with no time lag, as predicted by “springs” at the ankle joint. The time shift between the COP and COM (Table 3) averaged ~4 ms for all conditions. A negative time shift difference means that COP lagged very slightly behind COM, which is what would be expected in a lightly damped system. Pure springs would predict COP to be exactly in phase with COM; a small damping in parallel with the springs would cause a small lag of COP behind COM.

The second prediction states that the sway would be proportional to \( K_e^{-0.5} \). As is evident from Fig. 6, this prediction was seen to be proportional to \( K_e^{-0.55} \), which was quite close even though the system was underdamped (as indicated by the bandwidth of our curve fits of the tuned mechanical system). Thus the springlike nature of the plantarflexors in the A/P direction and the hip abd/adductors in the M/L direction represents a simple 0th order feedback control. The role of the CNS in this balance control appears to be to set the muscle tone such that the spring constant, \( K_e \), is sufficiently large to overcome the gravitational load (Wh) and to cause COP to move more than COM. In the A/P direction, subjects routinely stand with the COM ~5 cm anterior of the ankle joint. Thus with the COP set to oscillate around 5 cm, the ankle plantarflexors moment for a 70-kg subject, for example, would be ~35 N·m. In generating this moment, the plantarflexors would have sufficient torque to generate a stiffness to cause the COP to move more than the COM when the pendulum sways. The effective spring constant, \( K_e \), averaged ~850 N·m/deg or ~15 N·m/rad, which means a restoring moment of 15 N·m is applied for every degree of rotation of COM. With the COM ~1 m above the ankle, a 1° error would be equivalent to 1.7 cm error in COP-COM. Similarly, in the M/L direction the COP will move in response to the muscle tone in the hip abductors/adductors. The M/L sway would cause the hip moments to change in phase with the sway, which will cause the unloading of one limb and instantaneous loading of the other (Winter et al. 1993, 1996). This load/unload mechanism will cause the COP to move laterally in advance of the COM. In the wider stance position the COP movement is more rapid because the base of support is wider and the same percentage change in the loading of each limb would cause a larger (and more rapid) movement of the COP between the feet. Thus the effective stiffness of the M/L balance control has increased, and the more rapid movement of the COP relative to the COM manifests itself in a higher \( f_e \) of the COP-COM error signal. Support for stiffness control of balance of parkinson-
nian subjects during quiet standing (Horak et al. 1996) was evident from increased muscle tone compared with elderly controls and parkinsonian subjects on levodopa. This increased stiffness resulted in a significantly decreased rate of COM forward velocity in response to sudden horizontal displacements of the support surface.

The third prediction demonstrates the variability of stiffness, $K_e$, over the 2-min standing period. The variability in $K_e$ was assessed for one subject standing in the 150 and 50% stance width over each 12.8 s of the 102.4-s record. Figure 7 shows the variation of $K_e$ at eight 12.8-s periods of the trial. This subject averaged $K_e = 7,530 \text{N} \cdot \text{m/rad}$ over the 102.4 s, but $K_e$ ranged from $\sim 4,000 \text{N} \cdot \text{m/rad}$ to almost $10,000 \text{N} \cdot \text{m/rad}$ during the entire trial for the 150% stance width condition. However, we see from the predicted sway versus stiffness (Fig. 3) and from the experimental curve (Fig. 6) that when $K_e$ is large, variations in $K_e$ result in small variations of COM amplitude. Across the 10 subjects the standard deviation of $K_e$ was very large (1,801 N · m/rad), but this resulted in only a small variability in COM (0.036 cm). However, for the 50% position, the curve has a much steeper slope such that small fluctuations in $K_e$ result in large fluctuations in COM amplitude. The variability in $K_e$ at this position (212 N · m/rad) was only 12% of that evident at the 150% stance width, but the variability in COM (0.108 cm) was three times that at the 150% stance width. The smaller variability in $K_e$ is also seen for one subject in the 50% stance width condition (Fig. 7). This means that the CNS control of muscle tone (stiffness) at the wide stance width can afford to be quite sloppy, whereas at the narrow stance width it must be more rigidly controlled. Thus wide stance widths would be recommended for balance challenged patients.

**Probability of reactive control during quiet standing**

The question now arises as to whether the literature supports a lack of reactive control using vision, vestibular, or somatosensory feedback. From our results (Table 3), there is essentially no difference between the eyes open and the eyes closed conditions during quiet standing. Thus vision does not appear to play a role in this quiet standing. The vestibular system, especially the otoliths, have the potential to measure the head’s horizontal acceleration in both A/P

![Figure 6. Experimental curve of COM displacement vs. $K_e$ for the 10 subjects in the M/L direction. Model theory predicted sway = $7.66 K_e^{-0.55}$. Experimental results show sway = $7.66 K_e^{-0.55}$. Variability of $K_e$ and sway for each stance width are also shown (1 SD). At the 150% stance width the variability in $K_e$ was quite large, but, because of the shape of the curve, the COM sway variability was low. However, at the 50% stance width, $K_e$ variability was small, which facilitates keeping the COM sway variability at reasonable level.](image1)

![Figure 7. Variability of $K_e$ for 1 subject standing at 150 and 50% stance widths. $K_e$ is seen to vary over each 12.8-s period over the entire record (102.4 s). At 150% stance width, the variability is very high compared with the variability at 50% stance width. This can be explained by the COM sway vs. $K_e$ curve (Fig. 6). A large change in $K_e$ at 150% stance width results in small changes in sway, whereas at 50% stance width, $K_e$ must be kept quite constant to keep sway variability low.](image2)
and M/L directions. However, the head accelerations in our subjects averaged between 1.6 and 1.8 cm/s² in the A/P direction and between 1.0 and 1.2 cm/s² in the M/L direction. These accelerations are less than the threshold of otolith sensation in humans (Benson et al. 1986) and vestibular neural response in cats (Lacour et al. 1978; Xerri et al. 1987). Also, the potential role of the otoliths would appear to be limited to estimating the head COM acceleration and not the total body COM. Finally, we could speculate that joint receptors have the potential to feed information to a COM estimator. Studies on humans during weight bearing have reported thresholds of joint receptors (Simoneau et al. 1996) and of vision, joint, and vestibular receptors (Fitzpatrick and McCloskey 1993). In the Simoneau et al. (1996) study at ankle angular velocities of \(0.75\text{s}^{-1}\), the movement perception thresholds ranged from 1.0 to 1.5°. At \(0.25\text{s}^{-1}\) the threshold increased and ranged from 1.7 to 2.0°. However, Fitzpatrick and McCloskey (1993) reported lower thresholds. Vestibular thresholds in A/P sway were \(0.6\text{s}^{-1}\) at \(0.35\text{s}^{-1}\), and ankle proprioceptive thresholds were \(0.17\text{s}^{-1}\) at \(0.06\text{s}^{-1}\). From our trials the A/P sway averaged \(\pm0.25\text{s}^{-1}\) with an average angular velocity of \(0.16\text{s}^{-1}\). In the M/L direction the angular sway ranged from \(0.05\text{s}^{-1}\) at 150% stance width to \(0.15\text{s}^{-1}\) at 50% stance width. The angular velocities in the frontal plane ranged from 0.05 to 0.11°/s. All these angular displacements and velocities are well below the thresholds in the A/P direction reported by Simoneau et al. (1996) but were slightly above the A/P proprioceptive thresholds reported by Fitzpatrick and McCloskey (1993). Konradsen et al. (1993) compared sway measures from seven subjects standing on one leg before and after the injection of a local anesthetic to the ankle joint. Also, they compared an active ankle rotation test (unloaded) and in both experiments they found no differences. These results suggest that joint receptors are below or just at the borderline of their thresholds to control during quiet standing. However, the laboratory-induced sway in the Fitzpatrick and McCloskey (1993) study do not fully replicate the conditions of quiet standing. Their laboratory conditions had a ramp perturbation with both displacement and velocity set. In natural sway the velocity is a maximum when the displacement is zero, and the velocity is zero when the displacement is maximum.

Finally, if we were to assume that the CNS continuously estimates the COM displacements in a reactive mode, we could estimate the delays in the motor response (as seen in the COP signal). Consider neural latencies (afferent delays) of 25 ms to a hard-wired COM estimator somewhere in the spinal cord, then another 25 ms efferent delay to the ankle or hip muscles followed by the low-frequency response of the ankle plantarflexors or hip adductors/adductors, as indicated by the muscle twitch second-order characteristics (Milner-Brown et al. 1973). The twitch times for the muscles were assumed to be between 53 ms for the hip muscles increasing to 106 ms for the plantarflexors. Thus the muscle can be modeled as a critically damped low-pass system with the cutoff frequency, \(f_c = 1/2\pi T\), where \(T\) is the twitch time. Computer modeling of such a reactive control would predict a net neuromuscular delay of between 150 and 260 ms of the COP behind the COM. Such delays were not seen, the average time differences between the COP and COM was 4 ms. We recognize that delays introduced by the low-pass filter characteristics of the muscle might be compensated for with appropriate high-pass filter in the feedback loop. But the afferent and efferent delays in the loop cannot be eliminated by feedback processing. These delays will introduce finite time difference between COP and COM if the system is operating in a reactive control mode.

Collins et al. (Collins and DeLuca 1993; Collins et al. 1995) claim that balance control is both open and closed loop. Their conclusions were based on the COP records alone as recorded with one force platform. Without any data on the COM, they would not be able to measure differences between the controlling variable and the controlled variable. Their more rapid (smaller) changes in the COP reflect the rapid higher frequency components of COP-COM, which directly relate to \(f_c\) and subsequent estimates of a stiffness constant \(K_e\). They claim these components to be closed-loop control, which would involve sensory feedback and a COP that would have considerable lag behind the COM. However, a passive open-loop control is now seen with COP virtually in phase with the COM.

Thus, based on the borderline sensory thresholds and afferent and efferent delay estimates, a reactive control would not be predicted in quiet standing.

**Potential advantage of stiffness control of upright posture during perturbed standing**

Such a stiffness mechanism could be important in response to unexpected external perturbations. Many researchers have reported latencies of \(\approx80\) ms in muscle activation to platform perturbations (cf. Horak and Nashner 1986). These latencies do not include the motor response time due to twitch response of the first recruited motor units, which would add further delay before the COP would start to move in the same direction as the COM. A stiffness control would act immediately as the joint angle changed, causing the COP to move in the same direction as the COM. Unfortunately, none of the research involving external perturbations has estimated the COM, and only a few have recorded COP changes. However, we would now predict an initial mechanical response of the COP to be in phase with the angular changes, which would then be augmented by the reflex response after the neuromuscular delays. This prediction has been confirmed experimentally (Little et al. 1997).

**Appendix**

Figure A1, A and B, shows the inverted pendulum model of the body in the sagittal and frontal planes.

In the sagittal plane model (Fig. A1A) consider the COM to be located a height, \(h\), above and a distance, \(x\), anterior of the ankle joints. The vertical ground reaction force, \(R_v\), has its COP located a distance, \(p\), anterior of the ankle joint. \(W\) is the body weight less the weight of the feet and equals the vertical reaction force, \(R\) at the ankles.

Consider the free body diagram of the foot where \(a_x\) is the vertical acceleration of the mass, \(m_x\), of the feet

\[
R + R_v - m_x g = m_x a_x
\]

But in quiet standing \(a_x = 0\)

\[
R_v = -R + m_x g
\]

The horizontal ground reaction force, \(R_v\), in quiet standing is <1N
and can be ignored. The sum of the moments acting at the ankle is
\[ M_a + R_p, \alpha - m_g x_a = I_a \dot{\alpha}, \]
But in quiet standing the angular acceleration of this foot, \( \dot{\alpha}_z \), is 0
\[ \therefore M_a = R_p, \alpha - m_g x_a = 0 \]
\[ M_a = R_p, \alpha \]
But
\[ m_g (x_a - p) = R_p, \alpha \]
\[ \therefore M_a = R_p, \alpha \]
Now consider the free body diagram of the inverted pendulum acting at the ankle joints
\[ M_a = W_x = I_a \alpha \]
\[ R_p, - W_x = I_a \alpha \]
where \( I_a \) is the moment of inertia of the body (without the feet) about the ankle joint and \( \alpha \) is the angular acceleration of the inverted pendulum.
For small angular sways
\[ \alpha \approx - \frac{\acute{g}}{h} \]
\[ R_p, - W_x = - \frac{I_a \dot{\alpha}}{h} \]
But \( R = W \)
\[ \therefore p_z - x = - \frac{I_a \dot{\alpha}}{W h} x \quad (A1) \]

In the frontal plane model (Fig. A1B), consider the mechanical closed loop consisting of the two lower limbs and the pelvis. The knees are considered to be locked. As in the sagittal plane, the sum of the two shear forces, \( \bar{R}_f + \bar{R}_p \), is \( \leq \) \( N \) and is ignored. The ankle reaction force acts at a distance \( p_z \) from the origin, and \( W \) acts a distance \( z \) from the origin. Four moments act on the closed loop system: \( M_w \) and \( M_a \) at the ankles and \( M_v \) and \( M_h \) at the hips. The total moment acting to control the closed loop, \( M_i \), is
\[ M_i = M_w + M_a + M_v + M_h \]
Or we can consider one single moment of force acting at the origin
\[ M_i = R_p, \alpha \]
This means that if the sum of the four moments at the joints equals zero, then the COP will lie exactly halfway between the ankles, or at the origin.

Consider the frontal plane inverted pendulum acting about the origin (Fig. A1B)
\[ R_p, - W_z = I_a \alpha \]
where \( I_a \) is the moment of inertia of the pendulum acting about the origin in the frontal plane and \( \alpha \) is the angular acceleration of the pendulum about the origin.
But
\[ R = W \quad \text{and} \quad \alpha \approx - \frac{\acute{g}}{h} \]
\[ \therefore p_z - x = - \frac{I_a \dot{\alpha}}{W h} x \quad (A2) \]
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