Fourier Analysis of Sinusoidally Driven Thalamocortical Relay Neurons and a Minimal Integrate-and-Fire-or-Burst Model

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Smith, Gregory D., Charles L. Cox, S. Murray Sherman, and John Rinzel. Fourier analysis of sinusoidally driven thalamocortical relay neurons and a minimal integrate-and-fire-or-burst model. J. Neurophysiol. 83: 588–610, 2000. We performed intracellular recordings of relay neurons from the lateral geniculate nucleus of a cat thalamic slice preparation. We measured responses during both tonic and burst firing modes to sinusoidal current injection and performed Fourier analysis on these responses. For comparison, we constructed a minimal “integrate-and-fire-or-burst” (IFB) neuron model that reproduces salient features of the relay cell responses. The IFB model is constrained to quantitatively fit our Fourier analysis of experimental relay neuron responses, including: the temporal tuning of the response in both tonic and burst modes, including a finding of low-pass and sometimes broadband behavior of tonic firing and band-pass characteristics during bursting, and the generally greater linearity of tonic compared with burst responses at low frequencies. In tonic mode, both experimental and theoretical responses display a frequency-dependent transition from massively superharmonic spiking to phase-locked superharmonic spiking near 3 Hz, followed by phase-locked subharmonic spiking at higher frequencies. Subharmonic and superharmonic burst responses also were observed experimentally. Characterizing the response properties of the “tuned” IFB model leads to insights regarding the observed stimulus dependence of burst versus tonic response mode in relay neurons. Furthermore the simplicity of the IFB model makes it a candidate for large scale network simulations of thalamic functioning.

In TRODUCTION

Considerable attention has been focused in recent years on the functioning of thalamic relays, because it has become clear that the thalamus does not serve as a simple, machine-like relay of information to cortex (for recent reviews, see Sherman and Guillery 1996, 1998). Instead the thalamus controls the extent and nature of information being relayed in a dynamic fashion that appears to be related to behavioral state and perhaps attentional demands. A good example is the lateral geniculate nucleus, the thalamic relay of retinal information to visual cortex. Only 5–10% of synapses on geniculate relay cells derive from retina. The rest derive from nonretinal sources, including local GABAergic cells, feedback afferents from visual cortex, and various pathways from the brain stem, and these modulate the nature of retinogeniculate transmission (Sherman and Guillery 1996, 1998).

In addition to the complexity of thalamic circuitry, the membrane properties of relay cells contribute to the nature of the relayed information. In particular, thalamic relay cells exhibit a voltage- and time-dependent, low-threshold, transient Ca2+ conductance, that, when activated, allows Ca2+ to enter the cell via T-type (for “transient”) Ca2+ channels, producing a transmembrane current, IT, and leading to a large depolarization known as the low-threshold Ca2+ spike. The inactivation state of IT determines whether information is relayed to cortex in tonic mode or burst mode (Jahnsen and Llinas 1984a,b; Sherman 1996). When the cell starts off relatively depolarized (above roughly −60 mV for >50–100 ms), IT is inactivated, and the relay cell responds to an excitatory input [e.g., a retinal excitatory postsynaptic potential (EPSP)] with sustained firing of unitary action potentials. This is the tonic firing mode. However, if the cell is hyperpolarized first (below roughly −65 mV for >50–100 ms), the inactivation of IT is removed (i.e., IT becomes deinactivated), and now a sufficient depolarization or EPSP will activate IT. The result is a low-threshold Ca2+ spike with a brief burst of 2–10 action potentials riding its crest.

One of the keys to understanding how thalamic relays work is to understand in more detail how the input/output properties of relay cells are affected by the inactivation state of IT. We sought to do this with both an experimental and modeling approach. By recording from relay cells of the lateral geniculate nucleus of the cat in vitro, we measured input/output properties by injecting into the cell sinusoidal currents that varied in amplitude, frequency, and mean level, and we performed analogous input/output experiments on a minimal relay cell model to test the degree to which the essential features of IT accounted for relay cell responses. In addition to providing an easily parameterized set of stimuli that lends itself to Fourier analysis, the use of sinusoidal current injection allows us to interpret our results in the context of the spatial and temporal frequency analysis paradigm that has such a successful history in visual systems neuroscience (for review, see Shapley and Lennie 1985). Of course, our use of Fourier techniques is not based on an assumption of the linearity of relay neuron responses but simply reflects an historically preferred method of extracting relevant measures of cellular response (see METHODS).

For the theoretical component of this study, we developed a minimal “integrate-and-fire-or-burst” (IFB) neuron model. This model is constructed by adding a slow variable representing the deinactivation level of IT to a classical integrate-and-
fire neuron model (Knight 1972). The IFB model is designed specifically to be as simple as possible while still quantitatively reproducing much of the empirically observed properties of the relay cells. One motivation for developing such a minimal model is to simplify the parameter selection process. Furthermore, because the IFB model is minimal, a detailed characterization of its response properties leads to insight regarding the stimulus dependence of burst versus tonic response modes in thalamic relay cells. A final motivation for development of the IFB model is to have a realistically tuned yet computationally undemanding relay cell model that can be used in large scale network simulations of thalamic function.

METHODS

Experimental methods

We performed intracellular recordings with the whole cell configuration on thalamic relay cells of young cats (5–8 wk of age) in compliance with approved animal protocols. We used a thalamic brain-slice preparation containing the lateral geniculate nucleus. Briefly, the animals were anesthetized deeply with 25 mg/kg ketamine and 1 mg/kg xylazine and a block of tissue containing the thalamic region was removed and placed in cold, oxygenated slicing solution containing (in mM) 2.5 KCl, 1.25 NaHPO₄, 10.0 MgCl₂, 0.5 CaCl₂, 26.0 NaHCO₃, 11.0 glucose, and 234.0 sucrose. Thalamic slices (250–300 µm) were cut in a coronal or sagittal plane with a vibrating tissue slicer and placed in a holding chamber (30°C) for ≥2 h before recording. Individual slices were transferred to a submersion-type recording chamber maintained at 30°C and continuously perfused with oxygenated physiological solution containing (in mM) 126.0 NaCl, 2.5 KCl, 1.25 NaHPO₄, 2.0 MgCl₂, 2.0 CaCl₂, 26.0 NaHCO₃, and 10.0 glucose, all at pH 7.4.

We used an Axoclamp 2A amplifier to obtain current-clamp recordings from geniculate relay neurons in the A-laminae, and we continuously monitored the bridge balance throughout the recordings. The recording pipette solution contained (in mM) 117.0 K-gluconate, 13.0 KCl, 1.0 MgCl₂, 0.07 CaCl₂, 0.1 EGTA, 10.0 HEPES, and 0.5% biocytin. Data were digitized, stored on-line using Axotape software (Axon Instruments), and also recorded onto VHS tape for off-line analysis. Current injection through the recording electrode consisted of a sinusoidal waveform with an AC component (Iₜ) that varied in both amplitude (50–800 pA) and frequency (0.1–100 Hz). The DC component (Iₒ) of the current waveform was altered to manipulate the firing mode of the neuron (i.e., burst vs. tonic). All experimental records of membrane potential of relay neuron in whole cell mode have been adjusted to account for a 10-mV junction potential.

Fourier analysis of experimental and theoretical records

Customized user M-files were written for MATLAB 5.2.0 (The MathWorks) to perform data analysis using an SGI Challenge supercomputer that runs the IRIX operating system. For each stimulus condition, a periodic histogram (qᵦ, k an integer, 0 < k < N–1, n = 64 bins) was constructed that tallied over c cycles of period T the number of action potentials (qᵦ) evoked by the experimental or model relay neuron at each of N blocks of phase relative to the applied current’s period. Accounting for the number of cycles recorded and the time represented by one bin of phase (i.e., for 64 bins, 1 bin is π/32 rad), we generated the (periodic) poststimulus response histogram (PSTH) defined by Qₜ = qᵦ/NcT. A discrete Fourier transform of this PSTH was performed, leading to a set of N complex valued numbers, Ẑₜ, given by (Press et al. 1992)

\[ \hat{Q}_n = \sum_{k=0}^{N-1} Q_k \exp(-2\pi i kn/N) \]  

each with an associated amplitude, Aₜ = |\hat{Q}_n|, and phase, Pₜ = arg(\hat{Q}_n)/2π. With the preceding definitions, \( F_0 = A_0/N \) has units of spikes/second and is the mean firing rate of the neuron; that is, \( F_0 = q_{tot}/cT \), where \( q_{tot} = \sum_k q_k \) is the total number of spikes during the trial. The fundamental or stimulus-driven component of the response, \( F_1 \), is given by \( F_1 = (A_1 + A_{N-1})/N = 2A_1/N \). (For the 2nd equality, we have used \( A_1 = A_{N-1} \), which follows because the raw histogram, \( q_1 \), is real valued). As defined in the preceding text, \( P_1 \) is the phase advance or lag of this stimulus-driven response, has units of cycles, and takes values between −0.5 and +0.5. In addition, we define the following "index of nonlinearity"

\[ \Gamma = \left[ \sum_{n=1}^{N-1} \frac{(A_n - 2A_1)^2}{\sum_{n=1}^{N-1} (A_n)^2} \right] \]  

This index of nonlinearity, \( \Gamma \), is the normalized power of all modulated (non-DC) components of the PSTH not accounted for by the fundamental component of the response. Thus \( \Gamma \) takes values between zero (completely linear) and one (completely nonlinear). \( F_0, F_1, P_1, \) and \( \Gamma \) are the response measures of relay cells on which we shall concentrate.

For clarity of figure presentation, we also normalized the raw histogram, \( q_1 \), by the total number of spikes during the trial (qₑtot). The result is a spike phase density histogram (SPDH), defined by \( p_k = q_k/q_{tot} \). This SPDH has unit area and represents the likelihood of observing an action potential at a particular phase of the applied current.

IFB model

GENERAL FEATURES. The IFB model is constructed by adding a slow variable to a classical integrate-and-fire model neuron. The slow variable, \( h \), represents the inactivation of the low-threshold Ca²⁺ conductance, which involves T-type Ca²⁺ channels and produces a transmembrane current, \( I_T \). The model equations are (Rinzel 1980)

\[ \frac{dV}{dt} = I_{app} - I_L - I_T \]  

\[ \frac{dh}{dt} = \begin{cases} -h/\tau_h & (V > V_h) \\ (1 - h)/\tau_h & (V < V_h) \end{cases} \]

The current balance equation, Eq. 3, includes the sinusoidal applied current, \( I_{app} = I_o + I_1 \cos(2\pi f t) \); a constant conductance leakage current \( I_L \) of the form, \( I_L = g_L(V - V_L) \); and the low-threshold Ca²⁺ current, \( I_T \). An action potential occurs whenever the membrane potential reaches the suitable firing threshold \( (V_f) \) such that \( V(t) = V_f \Rightarrow V(t^+) = V_{reset} \). Equation 4 is an idealization of the dynamics of \( I_T \). The deactivation level of \( I_T, h \), relaxes to zero with time constant \( \tau_h = 20 \text{ ms} \) when \( V > V_f \) and relaxes to unity with time constant of \( \tau_h = 100 \text{ ms} \) when \( V < V_f \). Sufficient hyperpolarization thus leads to increasing values of \( h \), which represents deactivation of \( I_T \). For simplicity, we choose the following form for \( I_T \)

\[ I_T = g_{m_{ike}}h(V - V_L) \]

where \( m_{ike} \) is a characterization of the activation of \( I_T \), \( m_{ike} = H(V - V_L) \), and \( H(\cdot) \) is the Heaviside step function. In this form, the IFB model is capable of postinhibitory rebound bursting. The parameter \( \tau_h \) sets the duration of the burst, whereas the parameter \( \tau_h \) sets the duration of hyperpolarization necessary to recruit a maximal postinhibitory rebound response.

In this form, the model is very simple—most of its behavior can be understood in terms of the dual thresholds, \( V_f \) and \( V_{reg} \), that are responsible for the activation of burst and tonic spiking, respectively. Because a moderate range of applied currents was used experimentally, we found that it was not necessary to include an absolute
refractory period in the spike generation mechanism. That is, in the experimental results presented here, the saturating regime of a relay neuron’s current-frequency relation rarely was sampled. Also for the sake of simplicity, we have not included in the model the hyperpolarization-activated cation conductance, \( I_h \) (also known as the “sag” current or \( I_{sag} \)).

We integrated Eqs. 3 and 4 using a 266 MHz LINUX workstation running XPP, an ordinary differential equation solver written by Bard Ermentrout at the University of Pittsburgh and available via the internet (http://www1.pitt.edu/~phase). All calculations were performed using the fourth-order Runge-Kutta integration method and a time step of 10–100 \( \mu s \).

**PARAMETER SELECTION.** Standard parameters were selected for the IFB model in the following fashion. First, experimental observations indicated that the resting membrane potential of the relay neurons recorded in vitro was \(-75\) to \(-65\) mV. In the absence of applied current (\( I_{app} \)), the leakage term (\( I_L \)) exclusively sets the resting potential of the neuron model. We thus set \( V_L \) to \(-65\) mV. A second experimental observation is that the relay neurons recorded in vitro are hyperpolarized sufficiently at rest so that \( I_h \) is deactivated.

Indeed, quiescent relay neurons in the slice responded to a brief depolarization with a burst of action potentials. For this reason, we set \( V_h \) to \(-60\) mV, ensuring that the threshold for activation (and deactivation) of \( I_T \) is several millivolts greater than \( V_L \), as observed experimentally. This value for \( V_h \) also roughly corresponds to the observed threshold for activation of bursts.

Although relay neurons from which we recorded were not physiologically identified as X or Y, we chose both the surface area (30,000 \( \mu m^2 \)) and capacitance (\( C = 2 \mu F/cm^2 \)) of relay neurons to be in agreement with measurements performed on physiologically identified relay cells in the cat’s lateral geniculate nucleus (Bloomfield et al. 1987). This value for \( S_A \) implies that a DC applied current of 300 pA corresponds to \( I_{sag} = I_h = 1\) \( \mu A/cm^2 \) in Eqs. 3. However, reported values of \( S_A \) are distributed over a range of 20,000–46,000 \( \mu m^2 \) for X relay cells and 33,000–55,000 \( \mu m^2 \) for Y relay cells (Bloomfield et al. 1987), and this could lead to a significant change in this correspondence.

With values for \( S_A \) and \( C \) fixed, the magnitude of the leakage conductance was chosen (\( g_L = 0.035\) mS/cm\(^2\)) so that the rheobase of the IFB model fell within the experimentally observed range of 250–400 pA. This value of \( g_L \) corresponds to an input resistance of 95 \( \Omega \), which also lies within the experimentally observed range.

Next, \( V_m = -35\) mV and \( V_{reset} = -50\) mV were chosen so that the value for \( V_m \) corresponded roughly to the observed tonic spiking threshold and the slope of the IFB model current-frequency relationship in tonic mode agreed with that of representative relay neurons. The time constants for deactivation (\( \tau_h = 100\) ms) and inactivation (\( \tau_a = 20\) ms) are taken from the experimental and theoretical literature (Coulter et al. 1989; Huguenard and McCormick 1992; Wang et al. 1995). Finally, the maximum conductance associated with \( I_T \) (\( g_T = 0.07\) mS/cm\(^2\)) was chosen so that the IFB model reproduces the experimentally observed maximum of \(-5-10\) spikes/burst for relay neurons in the slice preparation.

**RESULTS**

Current-clamp recordings from a total of 12 relay neurons from the lateral geniculate nucleus of the cat were included in the present study. All cells displayed physiological characteristics consistent with healthy relay neurons, including a hyperpolarization-activated sag current (\( I_h \)), burst discharge, and overshooting action potentials. Cells included in this study had an average resting membrane resting potential of \(-67.8 \pm 4.1\) mV (mean \( \pm \) SD) and an input resistance that averaged 153.0 \( \pm \) 38.0 \( \Omega \).

The preferred firing mode of the relay cell—tonic or burst—was controlled largely by constant current injection from the recording pipette. This constant current injection is referred to in the following text as \( I_0 \) which we varied between experiments from \(-400\) to \(800\) pA. On top of this constant current, we injected various other currents, usually sinusoidal at various frequencies and amplitudes. At relatively depolarized membrane potentials (i.e., more positive values of \( I_0 \)), the cell fired to the sinusoidal current in tonic mode, whereas at relatively hyperpolarized membrane potentials (i.e., more negative values of \( I_0 \)), the cell fired in burst mode; at intermediate levels of membrane potential, responses often consisted of a burst followed by tonic firing. It is noteworthy that in all cases when we saw both response modes to a cycle of the current injection, burst firing always preceded tonic firing.

**General responses to current injection**

In Fig. 1A, the results of current steps injected into the relay cell experimentally are shown. When the membrane potential (\( V_m \)) was adjusted initially to \(-58\) mV (\( V_{hold} \), \( I_0 = 230\) pA), the neuron discharged in tonic mode in response to a short current pulse (200 pA). However, at a more hyperpolarized \( V_{hold} \) (\(-77\) mV), a depolarizing current step (50 pA) evoked a transient burst of high-frequency action potentials. The IFB model produces both tonic and burst responses (Fig. 1B) that are similar to those produced experimentally. Figure 1B also shows the inactivation gating variable of the simulated low-threshold \( \text{Ca}^{2+} \) current (denoted by \( h \); see METHODS) dropping from unity to near zero shortly after the onset of the current pulse. In the IFB model, the time scale of this inactivation determines the length of the burst event. The value of \( h \) remains near zero (representing inactivation of the low-threshold \( \text{Ca}^{2+} \) conductance) until the depolarizing pulse ends, at which point the membrane potential drops below \( V_h \), the threshold for deactivation of \( I_T \), and \( h \) recovers toward 1.

Figure 2A presents experimental recordings that illustrate firing patterns of a geniculate relay neuron during sinusoidal current injection over a range of stimulation amplitudes (\( I_1 \)) while the modulation frequency is held constant at 1 Hz. In Fig. 2A, left, the neuron is in tonic mode, because of a more hyperpolarized \( I_0 \) of 335 to 498 pA, whereas in the right column, the cell is hyperpolarized because of a more hyperpolarized \( I_0 \) of \(-4\) to \(-136\) pA and responds with burst discharges. When the neuron responds in tonic mode, the average number of spikes/cycle increases as \( I_1 \) is increased. In contrast, in burst mode, the neuron responds with 1 burst/cycle over a wide range of \( I_1 \), and the number of spikes/burst remains relatively constant at 7 or 8. An exception to this occurs at the lowest-modulation amplitudes, where the neuron bursts once for every several cycles of applied current (Fig. 2A, top right). In this paper, we will refer to such behavior as a subharmonic burst response. In addition to these representative patterns of cell responses to 1 Hz stimulation, we also observed superharmonic burst responses (2 bursts/cycle), subharmonic tonic responses, and burst followed by tonic responses.

Figure 2B presents a series of IFB model calculations for comparison with Fig. 2A. These calculations were performed using identical cellular parameters (see Table 1), whereas applied current parameters were chosen in qualitative agreement with the experimental conditions used in Fig. 2A. The
simulations reproduce many salient features of the experimen-
tal recordings. For example, the IFB model responds in tonic
mode to more depolarizing mean applied current ($I_0$) and in
burst mode to more hyperpolarizing $I_0$. In tonic mode, the
average number of spikes/cycle increases as a function of
$I_1$, whereas in burst mode, 1 burst/cycle is observed over a wide
range of $I_1$. In addition, the IFB model produces 7–8 spikes/burst, relatively independent of
$I_1$. Although the IFB model reproduces the overall pattern of responses to a range of stimulus conditions in both tonic and burst responses, it does
not reproduce the subharmonic responses observed at low
modulation amplitude (cf. Fig. 2, A and B, top right; see DISCUSSION).

Figure 3A consists of recordings from the same neuron as
Fig. 2A, but here sinusoidal current injection over a range of $I_0$
values and stimulation frequencies is applied while $I_1$ is fixed.
When the holding $V_m$ is adjusted so that the neuron is in a tonic
firing mode (depolarized $I_0$; left), the cell exhibits tonic firing
in response to $I_1$ frequencies of 0.3, 1, and 3 Hz, and is
unresponsive to 10 Hz. The average number of spikes/cycle
decreases in tonic mode as frequency increases. However,
when the neuron is in burst mode (hyperpolarized $I_0$; right), 1
burst/cycle (5–8 spikes/burst) is observed in response to 0.3
and 1 Hz until subharmonic responses are evoked at 3 Hz; no
response was seen at 10 Hz. The subharmonic responses were
observed most commonly in response to a frequency of 3 Hz,
occasionally at 1 Hz. Many other cells did respond to 10 Hz
stimulation, especially in tonic mode (see following text). In 4
of 12 cells, superharmonic responses of 2 bursts/cycle were
observed at low stimulation frequencies (0.1–0.3 Hz; see fol-
lowing text).

Figure 3B presents a series of IFB model calculations that
reproduce many salient features of the experimental results
shown in Fig. 3A. For example, when the model responds in
tonic mode (left), the average number of spikes per cycle
decreases as frequency increases. Yet when the model re-
sponds in burst mode, 1 burst/cycle is observed over a wide
range of frequencies of $I_1$. At the cutoff frequency of 10 Hz,
both the relay neuron and the IFB model are unresponsive, and
the effect of the mean applied current, $I_0$, on the mean mem-
brane potential can be seen clearly (see Fig. 3, bottom). The
IFB model thus qualitatively reproduces the neuron responses
over a range of stimulus frequencies with the exception of
subharmonic responses.

Superharmonic burst responses

As mentioned in the preceding text, burst responses ob-
served at low frequency were primarily 1 burst/cycle (1:1).
However, superharmonic burst responses were sometimes ob-
served at low frequencies. Examples recorded from two geniculate relay cells of single (1:1) and double (2:1) burst responses at low frequency are shown in Fig. 4, A and B, respectively. Figure 4, left, shows responses to a more hyperpolarized $I_0$, and the right shows responses to a more depolarized $I_0$ (see legend for details). The result is pure burst responses on the left and burst followed by tonic responses on the right. Because superharmonic burst responses were seen in response to the lowest temporal frequencies tested, which was 0.1 Hz, the prevalence of these two response types in our data have been quantified in the following way. There were a total of 56 trials collected from 12 different cells that exhibited bursts at 1–3 Hz. Of these, we quantified the response type at the lowest frequency tested, 0.1 Hz. The majority (61%) were 1:1 (i.e., 1 burst/cycle as in Fig. 4A). Superharmonic bursts (i.e., 2:1 as in Fig. 4B) were observed in 14% of the trials, whereas 3 bursts/cycle (3:1) were never observed. Interestingly, in the remaining 25% of trials, no response at all was observed at 0.1 Hz. However, at a higher frequency (0.3 Hz), only 3 of 56 trials (5%) exhibited no response, indicating that burst mode for some neurons has a genuine band-pass character to varying temporal frequency (see following text).

### TABLE 1. Standard parameters for the IFB model

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<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Unit</th>
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<tr>
<td>$V_u$</td>
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<td>mV</td>
</tr>
<tr>
<td>$V_L$</td>
<td>$-65$</td>
<td>mV</td>
</tr>
<tr>
<td>$C$</td>
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<td>μF/cm²</td>
</tr>
<tr>
<td>$g_{Lr}$</td>
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<td>mS/cm²</td>
</tr>
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<td>$V_{reset}$</td>
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<td>mV</td>
</tr>
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<td>mV</td>
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<td>mV</td>
</tr>
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<td>ms</td>
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</tr>
<tr>
<td>$f$</td>
<td>0.1–100</td>
<td>Hz</td>
</tr>
</tbody>
</table>

IFB, integrate and fire or burst.

Responses as a function of frequency and amplitude of current injection

The filtering characteristics of relay neurons differ depending on the firing mode of the neuron. Figure 5 summarizes responses of two relay cells in both burst (hyperpolarized $I_0$) and tonic mode (depolarized $I_0$) to different levels of $I_1$ and frequency of current injection. The responses have been categorized into the following four classes: burst followed by tonic...
(filled circles), only burst (open circles, left) or only tonic (open circles, right), subharmonic burst (asterisks, left) or subharmonic tonic response (asterisks, right), and no response (dashes). Figure 5A, left, summarizes the responses of one of these neurons in burst mode. At low $I_0$ (50 pA), this neuron responded to 3 Hz with subharmonic bursts (asterisks) and gave no responses to higher or lower frequencies. With increasing $I_1$ (100 pA), the cell now responds 1:1 at 0.3 and 1 Hz, subharmonic at 3 Hz, but is still unresponsive to 0 Hz. Thus burst responses of this cell show band-pass filtering for lower $I_0$ and low-pass filtering for higher $I_0$. In this example, no high-frequency cutoff is observed because $I_0$ was greater than the rheobase of neuron, whereas in the other example (Fig. 5B, right), $I_0$ was less than the rheobase. It is also notable that subharmonics in tonic mode, when apparent, generally occurred before the cutoff frequency.

Fourier analysis of relay cell and IFB model responses

To compare quantitatively the different consequences of burst and tonic firing modes on relay cell responses to sinusoidal current injection, we performed Fourier analysis of the intracellular recordings (see METHODS). SPDHs were constructed from experimental recordings, and Fig. 6 shows examples of how this is done for several different stimulus conditions producing burst, tonic, or burst followed by tonic firing (see following text for details of stimulation parameters). Figure 6A shows responses to four cycles of the injected current. Figure 6B shows these responses aligned on a cycle-by-cycle basis, and Fig. 6C shows SPDHs constructed by assigning each spike to 1 of 64 bins depending on the value of its phase with respect to $I_1$. In the resulting histograms, the spike density, $r$, approximates the likelihood of a neuron firing an action potential at a given phase, $\phi$, of $I_{app}$. To quantify the dependence of the SPDHs on stimulus parameters ($f$, $I_0$, and $I_1$), discrete Fourier transforms of such histograms were performed leading to the assignment of four response measures:
**Figs. 4, 5, and 6.** Representative responses for relay neurons driven at low frequencies. Asterisks indicate bursts. A: at 0.1 Hz, this relay neuron responds with 1:1 bursting (i.e., 1 burst/cycle). B: different relay neuron responds to 0.1-Hz stimulation with 2:1 superharmonic bursting (i.e., 2 bursts/cycle). In both cases, increasing \(I_o, I_t\), or both leads to the recruitment of tonic spikes and the generation of a burst followed by tonic (burst-tonic) response. \(I_o/100\) in pA for the burst and the burst followed by tonic firing case, respectively: A, 0/300, 130/300; B, 60/150, 170/240.

\[ F_0, \text{the mean firing rate}; F_1, \text{the stimulus- or modulation-driven component of the response}; P_1, \text{the phase advance or lag of the modulation-driven response}; \text{and } \Gamma, \text{the nonlinearity of the response.} \]

Figure 6C shows examples of SPDHs that are representative of our results from relay cells exhibiting tonic, burst, and burst followed by tonic responses obtained at 0.3 Hz. When stimulus parameters were such that the neuron responded in tonic mode (\(I_o = 410 \text{ pA and } I_t = 150 \text{ pA}\)), the SPDH approximates the shape of a rectified cosine. When stimulus parameters were such that the neuron responded in burst mode (\(I_o = -4 \text{ pA and } I_t = 50 \text{ pA}\)), the SPDH does not approximate a rectified cosine but rather shows a sharp peak near \(\phi = -0.25\) (i.e., a 90° phase advance). These features are combined in a SPDH generated from intracellular records that exhibited burst followed by tonic responses (Fig. 6C). The burst and tonic portions of this response are separated by \(\approx 10^2\), and because the burst always proceeds the tonic response of each cycle, there is a gap in the SPDH.

**INTERPRETATION OF RESPONSE MEASURES.** Because many of the results to follow involve the relationship between the response measures, \(F_0, F_1, P_1, \text{and } \Gamma\), it is instructive to review our expectations of these measures. Imagine an idealized tonic response of a neuron to be proportional to the rectified cosine (see Fig. 6C), most commonly

\[ Q_{\text{rec}}(\phi) = \frac{1}{2} Q_{\text{rec}}^{\text{max}} H(1 - 2R + \cos(2\pi\phi))[1 - 2R + \cos(2\pi\phi)] \]

\[ (0 < R < 1) \]  

(6)

where \(R\) indicates the degree of rectification, \(H(\cdot)\) is the Heaviside step function, and the maximum response is given by

\[ Q_{\text{rec}}^{\text{max}} = Q_{\text{rec}}(1 - R) \]

(7)

Figure 7E shows examples of SPDHs of this form and Fig. 7, A and B, shows the resulting dependence of the response measures \(F_0\) and \(F_1\) (open circles with solid line) on the degree of rectification, \(R\). As expected, both \(F_0\) and \(F_1\) are decreasing functions of \(R\), because rectification decreases both the mean firing rate and modulation-driven component of the rectified cosine response. Figure 7C shows for tonic firing the quotient \(F_1/F_0\) as a function of \(R\). This plot indicates that a sinusoidal response with no rectification (and a mean equal to its modulation amplitude) will have \(F_1/F_0 = 1\). Conversely, highly rectified responses will have \(F_1\) approximately twice as large as \(F_0\). For comparison, Fig. 7F shows PSTHs similar to those we observed from burst responses. These histograms are square pulses of the form

\[ Q_{\text{sp}}(\phi) = \begin{cases} Q_{\text{sp}}^0 & \text{for } -(1 - R)/2 < \phi < (1 - R)/2 \\ 0 & \text{otherwise} \end{cases} \]

(8)

Figure 7, A and B, shows that the dependence of \(F_0\) and \(F_1\) on the degree of rectification, \(R\), is different for square pulses and rectified cosines. For example, in the case of square pulse responses (open squares with solid line), \(F_0\) decreases linearly and \(F_1\) is nonmonotonic, reaching a maximum when the square pulse occupies one-half of the stimulus cycle.

Figure 7D shows the index of nonlinearity (\(\Gamma\)) expected for square pulse and rectified cosine responses, respectively. When a sinusoidal response shows no rectification (\(R = 0\), leftmost open circle), the index of nonlinearity (\(\Gamma\)) is zero, because the power in \(F_0\) and \(F_1\) account for the entire histogram. Although an extremely high degree of rectification (\(R = 1\)) can lead to an index of nonlinearity near unity, the tonic responses we observed experimentally were always less than three-fourths rectified (cf. Fig. 7E). We thus expect, according to the plot of \(\Gamma\)
in Fig. 7D, that low-frequency tonic responses will generally have an index of nonlinearity <0.3 (open circles with solid line). Similarly, Fig. 7D shows that the index of nonlinearity \( \Gamma \) of square pulse responses can be large or small depending on \( R \) (dotted and solid lines with open squares). However, because experimentally observed SPDHs generated from burst responses always had \( R \) values >0.85 (solid lines), the indices of nonlinearity observed were also high \( \Gamma > 0.7 \). Thus during low-frequency stimulation, when relay cell responses resemble the histograms shown in Fig. 7, \( E \) and \( F \), we expect the index of nonlinearity, \( \Gamma \), to be highly correlated with the burstiness of the response.

FOURIER ANALYSIS OF EXPERIMENTAL DATA. With this background, Figs. 8 and 9 present the results of Fourier analysis of the responses of a population of relay cells showing burst and tonic responses to sinusoidal current injection. Only data involving pure burst or tonic responses (i.e., no burst followed by tonic responses) are shown here, which means that these data reflect mostly small values of \( I_t \) and/or extremely hyperpolarized or depolarized values of \( I_p \). Figure 8, A–D, summarizes experimental results from 84 trials (8 different cells) in which only burst responses were observed. When plotted in units of spikes/second, the mean firing rate \( (F_0) \) is approximately half the value of the modulated response \( (F_1) \), and both are band-pass with peaks at 3 Hz (Fig. 8A). These data subsequently are plotted in units of spikes/cycle (Fig. 8B), showing that, at lower frequencies, a relatively constant number of spikes/cycle are observed, and in the majority of cases, these result from one burst/cycle.

Because the response in burst mode is focused at a particular phase at low frequencies, the index of nonlinearity \( \Gamma \) of burst responses is high for all injection frequencies that elicit a response (see Fig. 8D). At 3 Hz each of the 64 bins of phase represents ~5 ms, as opposed to 150 ms in the 0.1 Hz case); thus in Fig. 8E, the...
3-Hz burst response appears less focused. This spread of the burst response as a function of phase causes \( G \) to drop slightly at 3 Hz.

Like most of the neurons from which we recorded, those illustrated in Fig. 8 did not respond at $10 \text{ Hz}$. However, this observed cutoff frequency does not necessarily correspond to the intrinsic upper limit for a relay cell in burst mode, because both experimentally (see Fig. 5A) and theoretically (see APPENDIX), the cutoff frequency is a function of the applied current parameters, $I_0$ and $I_1$. A small value of $I_1$, for example, often leads to a lower cutoff frequency (see Fig. 5).

Fourier analysis of relay cell responses to sinusoidal current injection in tonic discharge mode shows a pattern distinct from that observed for burst firing mode. Figure 9, A–D, summarizes experimental data from 88 trials (10 different cells) in which only tonic responses were observed. Similar to burst responses, during tonic firing, $F_0$ was generally less than $F_1$ (Fig. 9, A and B). The responses of these neurons in spikes/second do not show a high-frequency cutoff (Fig. 9A) because, for the majority of neurons included in this sample, $I_0$ is superthreshold for firing action potentials (cf. Fig. 5A). Figure 9B shows that when these responses are replotted in spikes/cycle, the number of spikes/cycle increases at lower frequencies, a pattern distinct from burst mode (cf. Fig. 8B).

The phase of tonic responses shows a gradual reduction with increasing stimulation frequency, from ~0.1 cycles advanced at 0.1 Hz and to a similar amount delayed at 100 Hz. The index of nonlinearity (\( \Gamma \)) of tonic responses reaches a maximum of 0.8 at 3 Hz, which is, coincidentally, a value comparable to that of burst responses at the same frequency. This might seem at odds with the predictions summarized in Fig. 7D. However, this elevated \( \Gamma \) is due to phase locking to 3-Hz stimulation that is apparent in the SPDHs presented in Fig. 9E. In this phase-locked superharmonic response, each of 3–4 spikes/cycle produces an identifiable peak in the SPDH. As a result of this phase locking, the response profile is considerably distorted from a rectified cosine. The significance of such phase locking for functioning of relay cells is considered in the following text and in DISCUSSION.
The SPDHs of Fig. 9E show a gradual transition from superharmonic tonic responses to phase-locked subharmonic tonic responses that is ubiquitous in our recordings and indicated by notation of the form, $S^C$, on the histograms (every $S$ spikes for every $C$ cycles). Superharmonic tonic responses (many spikes/cycle) are observed when the stimulating frequency is low (0.1–1 Hz). At a frequency of 3 Hz, phase-locked superharmonic spiking (3:1 to 4:1) begins, whereas at slightly higher frequencies (10 Hz), the neuron exhibits phase-locked action potentials exactly once per cycle (1:1). At higher frequencies of 30–100 Hz, the neuron responds with subharmonic tonic spikes (1:2 to 1:14) that are approximately phase locked to the applied current. Although the highest frequency at which 1:1 tonic spiking is observed is 10 Hz (see Fig. 9E), this does not reflect an intrinsic limit but rather depends on the particular values of $I_0$ and $I_1$ used (see Discussion and Appendix).

Comparing the frequency-dependence of $\Gamma$ in burst (Fig. 8D) and tonic (Fig. 9D) mode, we observe that at frequencies <1 Hz, $\Gamma$ correlates with the burstiness of the response. However, at higher frequencies, the phase locking of tonic responses leads to elevated $\Gamma$ that is distinct from nonlinearity arising from rectification (Fig. 7D). We also observe that at 0.1 Hz, $\Gamma$ exceeds 0.16, the value expected for a half-wave rectified but otherwise linear response (as shown in Fig. 7D).

FOURIER ANALYSIS OF THE IFB MODEL. Figures 10 and 11 present the results of Fourier analysis applied to IFB model responses. As in Figs. 8 and 9, $I_0$ and $I_1$ are chosen to ensure pure tonic or burst responses from the model (burst followed by tonic responses are considered in the following text). For burst firing, there is a remarkable, quantitative equivalence between experiment (Fig. 8) and theory (Fig. 10). One subtle difference is that the IFB model phase locks more precisely than actual relay cells (cf. 3-Hz SPDHs in Figs. 8E and 10E). This may be due to the presence of noise in the voltage recordings that is absent in the IFB model. Indeed, when the simulated applied current is supplemented with Gaussian noise (mean amplitude of 0 pA and a variance similar to that of $I_1$), phase locking by the IFB model is strongly attenuated (data not illustrated), and the nonlinearity index ($\Gamma$) is reduced. IFB
A model tonic responses (Fig. 11) are also comparable with experimental records (Fig. 9), though a notable exception is a gradual drop in experimental records (Fig. 9), though a notable exception is a model tonic responses (Fig. 11) are also comparable with experimental records (Fig. 9), though a notable exception is a gradual drop in experimental records (Fig. 9), though a notable exception is a transition from superharmonic to subharmonic spiking that is qualitatively similar to that seen experimentally (cf. Figs. 9E and 11E). At low frequencies, the IFB model responds with superharmonic tonic spikes (many spikes/cycle). At 3 and 10 Hz, the IFB model responds in a phase-locked fashion (4:1 and 1:1, respectively). At 30–100 Hz, phase-locked subharmonics responses are produced. Because of the strong phase-locking properties of the IFB model (Keener et al. 1981), the theoretical SPDHs in this frequency range are more focused than the experimental SPDHs. This causes $\Gamma$ to be elevated compared with experiment in the 30- to 100-Hz range (cf. Figs. 9D and 11D).

**Phase plane portrait of the IFB model and high-frequency roll off in burst mode**

As the cutoff frequency is approached, burst responses of relay cells gradually decline or roll off (e.g., Fig. 8, A and B). One obvious reason for this is that some neurons exhibit subharmonic bursts in this frequency range, and this would cause a decrease in spikes/second and spikes/cycle. In a subset of trials that exhibited bursting in the range of 1–3 Hz, we found subharmonic (1:N) bursting and 1:1 bursting to be nearly equally prevalent at 3 Hz (36 and 43%, respectively), suggesting for some cells another reason for this roll off, one that is predicted by our IFB model. Figure 12A shows burst responses from the IFB model that demonstrate the roll off in $F_0$ during 1:1 bursting. At 2 Hz, the model responds with 1 burst/cycle, and each burst is composed of 6 spikes/burst. However, at 6 Hz, where 1 burst/cycle also is seen, only 2 spikes/burst are evoked. Also note that the maximum deactivation levels ($h_{\text{max}}$) of $I_T$ achieved during the hyperpolarizing phase of the applied current is much greater at 2 than 6 Hz (Fig. 12B). The greater $h_{\text{max}}$ at 2 Hz thus leads to a larger evoked low-threshold Ca$^{2+}$ spike, which, in turn, evokes more action potentials.

Figure 12C presents phase-plane portraits of the IFB model...
at 2 and 6 Hz. This helps to clarify the relationship between membrane potential \( V \) and thus spike discharge, and the inactivation gating variable, \( h \), at both frequencies. The solid line and arrows show the trajectory in the \((V, h)\) plane that is repeated from cycle to cycle. The threshold for \( I_T \) \((V_h, \text{dashed line})\), \( V_{\text{reset}} \) \((\text{dot-dashed line})\), and \( V_u \) \((\text{dotted line})\) also are indicated. During the hyperpolarizing phase of the applied current, \( V \) eventually drops below \( V_h \), causing \( h \) to increase (arrow 1). Eventually the current reverses, leading to depolarization; however, because \( V \) is still less than \( V_h \), \( h \) continues to increase (arrow 2). When the membrane potential crosses \( V_h \), \( I_T \) activates, \( h \) begins to drop, and \( I_T \) depolarizes the model neuron until the spike threshold, \( V_{ip} \), is reached (arrow 3). A series of action potentials are evoked (arrow 4) and \( h \) decreases until the sum of \( I_T \) and the applied current are no longer large enough to bring the membrane potential above threshold. When the applied current again reverses, the membrane potential hyperpolarizes, \( V \) eventually drops below \( V_{ip} \), and the periodic burst response repeats.

Figure 12D shows a plot of the frequency-dependence of \( h_{\text{max}} \). Because the time constant for inactivation of \( I_T \) is smaller than the time constant for its deinactivation (see METHODS), \( h_{\text{max}} \) decreases as frequency increases. This, in turn, means that the size of the evoked low-threshold \( \text{Ca}^{2+} \) spike and the number of action potentials riding its crest will decrease at higher frequencies. It is thus this decline in \( h_{\text{max}} \) as a function of frequency that leads to the high-frequency roll off in the IFB model burst response. Although we do not have direct access to the gating variable, \( h \), in our experimental recordings, the open squares and diamonds in Fig. 12E show the number of spikes/burst exhibited by two relay neurons that burst 1:1 at all frequencies tested. This qualitatively matches roll off in spikes/burst exhibited by the IFB model (open circles) using standard parameters, although a better fit for these particular trials was obtained by increasing \( t_h \) from 100 to 300 ms, which resulted in a lower cutoff frequency (filled circles).

Dependence of response measures on modulation amplitude

In the analyses summarized in Figs. 8–11, \( I_1 \) was fixed and small enough to avoid burst followed by tonic responses, and \( I_0 \) controlled the response mode by being either relatively depolarizing (for tonic firing) or hyperpolarizing (for burst firing). However, in both modes, the quantitative value of response measures depends on \( I_0 \) and \( I_1 \). This is true for both actual relay cells as well as the IFB model. Figure 13, A–C, left, presents the frequency dependence of the fundamental response \( F_1 \) for 11 relay neurons using either low \( I_1 \) (50–200 pA;
filled squares) or high $I_1$ (300–500 pA; open triangles), and the right panels show the comparable responses from the IFB model. These results are pooled according to the firing mode based on $I_0$ so that Fig. 13A ($I_0$ high) presents responses that are predominantly tonic, whereas Fig. 13C ($I_0$ low) presents predominantly burst responses, and Fig. 13B ($I_0$ medium) includes many burst followed by tonic responses.

Note that the fundamental response is generally greater in Fig. 13 for the higher $I_1$ than for the lower regardless of response mode. During tonic firing (Fig. 13A), there is little if any frequency dependence in the response augmentation with higher $I_1$. However, when bursting occurs (Fig. 13, B and C) the degree of augmentation for large $I_1$ can be frequency dependent. For example, the experimental responses show a greater $F_1$ for the larger $I_1$ at lower frequencies, but there is little difference in $F_1$ at 10–100 Hz (note the overlapping error bars in Fig. 13B, left, for these frequencies). Qualitatively, this pattern is matched in the IFB model (Fig. 13B, right), which shows a clear peak in the augmentation of $F_1$ with $I_1$ at 3 Hz and no difference in response to higher frequencies. In experimental observations of burst mode (Fig. 13C, left), there is a clear distinction between low and high $I_1$.

Because with the fairly stable number of spikes/burst we observe, the response in spikes/seconds becomes increasingly small at lower frequencies. The IFB model qualitatively exhibits these same features (Fig. 13C, right), and the response function for $F_1$ reproduces this band-pass characteristic. However, for the most part this attenuation is “artificial,” that is, it simply reflects the fact that the response is measured in spikes/second. To clarify this point, the solid squares in Fig. 13D redisplay in units of spikes/cycle $F_1$ for the low $I_1$ case shown previously (Fig. 13C, solid squares). In these units the neuronal response is low-pass rather than band-pass (cf. Fig. 8, A and B).

At the lowest values of $I_1$ tested, we found that 6 of 11 neurons were unresponsive to the lowest frequencies tested but responsive to higher ones (see Fig. 5), while the remaining 5 responded to even the lowest frequencies. The open circles in Fig. 13D show the average $F_1$ in units of spike/cycle. The $F_1$ is attenuated at 0.1 Hz relative to the 0.3- and 3-Hz response, even when measured in units of spike/cycle (Fig. 13D, open circles). This occurs because approximately half of the trials were genuinely band-pass, that is, for this low value of $I_1$, 50% of the neurons did not respond at 0.1 Hz even though they did respond with 1 burst/cycle at 0.3 Hz.

**Parameter studies of IFB model responses**

To give a sense of the dynamics of transitions between tonic and burst modes of firing as parameters vary, Fig. 14, A–C, presents a series of raster plots calculated using the IFB model in which $I_0$ is varied while the frequency and cellular parameters are fixed, and amplitude of modulation increases from A to C (see legend of Fig. 14 for details). In Fig. 14A, where $I_1$ is low, as $I_0$ becomes more depolarizing, one sees a transition from silence to burst responses to another silent zone and then to tonic responses. Another transition in Fig. 14A is observed near $I_0 = 1.4 \mu$A/cm$^2$, where tonic spikes occur throughout the stimulus cycle (as opposed to the interrupted spiking). This happens when $I_0$ is sufficiently depolarizing to ensure that, when added to $I_1$, the model cell’s membrane potential is always continuously driven above threshold for action potentials. The open circles in Fig. 14A show the phase of the centroid of the response ($P_1$) plotted relative to the time of an applied current maximum (vertical dot-dashed line). Burst responses are phase lagged when first recruited at low $I_0$ but then advance in phase as $I_0$ is increased. Tonic responses evoked as $I_0$ becomes more depolarizing consistently have a phase near zero.

By allowing a close look at spike timing as $I_0$ is varied, the raster plot of Fig. 14A complements the Fourier analysis presented previously. For example, the phase of individual tonic
spikes generally advances with increasing $I_0$. The new spikes that occasionally are recruited as $I_0$ increases generally appear in the late phase of the stimulus cycle, that is, when the applied current is decreasing. During interrupted tonic spiking, there is no consistent increase in $P_1$ as a function of $I_0$ because the gradual phase advance of individual spikes is balanced by new spikes arriving later in the cycle.

The gap between burst and tonic responses in Fig. 14A indicates a range of $I_0$ for which the IFB model does not respond. Such a gap also was observed experimentally (data not shown) and is present in the IFB model when $I_1$ is small enough so that, over a certain range of $I_0$, the membrane potential oscillates in response to the applied current without crossing either of the two thresholds, $V_u$ or $V_h$. This gap is reduced or nonexistent with larger values of $I_1$ (Fig. 14, B and C). Figure 14B shows the result of larger $I_1$, and here burst responses appear more robust, beginning at lower values of $I_0$ and extending into higher values. Comparing Fig. 14A and 14B, one also sees that increased $I_1$ extends the range of interrupted tonic firing to higher levels of $I_0$. In Fig. 14C, $I_1$ is made even larger, leading to burst followed by tonic responses over a large range of $I_0$. The transition point between interrupted and continuous tonic spiking has moved upward beyond $I_0 = 3.0 \mu A/cm^2$ and is no longer visible.

Figure 14, D–F, shows rastergrams of the IFB model burst responses as a function of the frequency of the applied current where the abscissa indicates the phase ($\phi$) of each spike. Because of the compression of time in this coordinate at lower frequencies, the burst events appear to narrow, even though their duration is relatively constant. Figure 14, D–F, varies only in $I_0$, which is increasingly depolarizing from D to F. Figure 14D thus illustrates only burst responses, and comparing the open circles and vertical dot-dashed line here reveals that the phase advance of these responses gradually changes to a phase delay as the frequency increases from 0.1 to 10 Hz (cf. Fig. 10C).

In Fig. 14, E is similar to D, but $I_0$ is depolarizing, and interrupted tonic responses are evoked. Between 0.1 and 1 Hz, the fraction of the stimulus cycle occupied by tonic spikes is approximately constant. The IFB model here is responding relatively linearly (e.g., $\Gamma = 0.15$ at 0.3 Hz; see Fig. 11D). From 0.1 to 6 Hz, $P_1$ is approximately zero (cf. Fig. 11C) even though the phase of the first tonic spike advances considerably in the range of 1–6 Hz. Above 6 Hz, the model responds with 1 spike/cycle, and $P_1$ follows the phase lag of this solitary spike until the cutoff frequency is reached. Here the response in nonlinear (e.g., $\Gamma = 0.94$ at 10 Hz; see Fig. 11D). The low-pass character (no response at $\geq 30$ Hz) of these tonic
responses reflects the fact that $I_0$ is subthreshold for action potentials here. For comparison, Fig. 14F presents a calculation identical to E except that $I_0$ is increased to a level that is superthreshold. In this case, the IFB model responds with tonic spikes even at 100 Hz. Strong phase locking is seen at higher frequencies with low $S:C$ ratios that change discontinuously with frequency.

The dual threshold character of the IFB model and the influence of $I_0$ on the functional dependence of $F_0$ and $F_1$ on $I_1$ is illustrated further in Fig. 15. With stimulation frequency fixed at 2 Hz, $I_0$ is more hyperpolarizing in the successive rows of Fig. 15, so that A exhibits purely tonic responses, E exhibits only burst responses, and B–D show gradual shift from pure tonic, to burst followed by tonic to pure burst responses. Voltage trajectories are presented for two representative values of $I_1$, the column labeled $I_1 Low$ presenting simulations with a modulation amplitude that is small in comparison with the $I_1 High$ column. These voltage trajectories are superimposed on dotted and dashed lines, indicating the tonic spike threshold ($V_u$) and $I_T$ threshold ($V_h$), respectively.

In Fig. 15, the arrows labeled $V_{ss}$ indicate the membrane potential that would be approached (in the absence of spiking) on the basis of the DC bias current alone so that $V_{ss} = V_L + I_0/g_L$, where $V_L$ is resting membrane potential and $g_L$ is leakage conductance of the model. Indeed, the relationship among $V_{ss}$, $V_u$, and $V_h$ determines the functional form of $F_0$ and $F_1$ as a function of $I_1$. For example, in Fig. 15A, $I_0$ is depolarizing enough that the steady-state voltage, $V_{ss}$, is greater than the threshold for action potentials ($V_{ss} > V_u$). Thus low $I_1$ gives continuous tonic spiking, whereas high $I_1$ gives interrupted tonic spiking. $F_0$ is thus nearly constant with $I_1$, whereas $F_1$ increases as a function of $I_1$. Compare this with Fig. 15B, where $I_0$ is less depolarizing, so that the steady-state voltage is near the action potential threshold ($V_{ss} = V_u$). In this case, low $I_1$ gives low-frequency interrupted tonic spiking, whereas high $I_1$ results in a burst followed by tonic response, because the

FIG. 12. Frequency dependence of $I_T$ inactivation. A: membrane potential time course for IFB model at $f = 2$ (left) and 6 Hz (right). B: time course of $h$, the inactivation gate of $I_T$, at 2 and 6 Hz. C: h vs. $V$ phase-plane portraits for the IFB model simulations shown in A and B. Dashed line (labeled $V_{reset}$), threshold for deinactivation (and activation) of $I_T$; dot-dashed line (labeled $V_{reset}$), value to which the membrane potential is set after each spike; dotted line (labeled $V_h$), action potential threshold. Arrows, direction of flow; numbers make correspondence between C and A. D: frequency-dependence of $h_{max}$, the maximum deinactivation level achieved during repetitive bursting. E: number of spikes/burst for both experimental observations and the IFB model. Shown are 2 representative relay neurons (open squares and diamonds), the IFB model with standard parameters (open circles), and the IFB model with $\tau_h = 300$ ms (filled circles). $I_0$ and $I_1$ in $\mu A/cm^2$: 0.0, 1.0.
minimum membrane potential during a cycle is now less than \( V_h \), thereby resulting in enough \( I_T \) deinactivation to allow the ensuing depolarizing half-cycle of the stimulus to evoke an early burst. Here \( F_1 \) is once again a nearly linear function of \( I_1 \); however, \( F_0 \) is no longer relatively constant as in Fig. 15A but rather has a similar slope as \( F_1 \). The voltage trajectory for the low \( I_1 \) case in Fig. 15C shows a null response followed after a threshold \( I_1 \) is exceeded by a fairly linear growth in both \( F_0 \) and \( F_1 \) with increasing \( I_1 \). Here \( V \) oscillates between \( V_u \) and \( V_h \) for low values of \( I_0 \) (see Fig. 15C, middle) and, consequently, the IFB model does not respond for this particular value of \( I_0 \), resulting in a gap between burst and tonic responses visualized previously in the IFB model raster plot (Fig. 14A). In Fig. 15D, \( I_0 \) is such that the steady-state voltage is near the burst threshold (\( V_{ss} = V_u \)) and a consequence of this is that even the lowest nonzero value of \( I_1 \) elicits a burst response. For Fig. 15E, the response is entirely composed of bursts, and \( F_0 \) and \( F_1 \) respond similarly with increasing \( I_1 \), showing a fairly sharp threshold when \( I_1 \) is sufficiently large to recruit bursts followed by a slight increase in firing.

The dependence of response measures on the value of both the mean and modulated applied current is summarized in Fig. 16, which presents Fourier analysis of IFB model responses to 0.3-Hz stimulation over a range of \( I_0 \) and \( I_1 \). Response mode is controlled by \( I_0 \) so that the majority of the responses in A are tonic, the majority of the responses in C are burst, and B includes a large number of burst followed by tonic responses. Filled squares, \( I_1 \) is low; open triangles, \( I_1 \) is high. Error bars indicate mean ± SE. D: average response (\( F_1 \)) per cycle as function of temporal frequency. Filled squares indicate data replotted from C, left. Open circles show a similar analysis when the lowest \( I_1 \) to which each cell responded was applied.
show that the response near this interface is nonlinear and phase advanced. Figure 16B plots the dependence of $F_1$ on $I_0$ and $I_1$, which follows the same general trends as indicated in Fig. 16A, the one major exception being that $F_1$ is much less than $F_0$ when $I_0$ is high and $I_1$ is low (bottom right of graph). The largest discrepancies between $F_0$ and $F_1$ occur when the IFB model is tonic spiking in a modulated, but continuous, manner. In Fig. 16D, the peaks of elevated $\Gamma$ that occur when $I_0 > 1.1 \mu A/cm^2$ and reflects phase locking that occurs when the firing frequency of the IFB model (determined solely by the DC current, $I_0$) are such that an integral number of spikes occur in one stimulus cycle. This phase locking is artifactual in the sense that when $I_1 = 0$ the stimulation frequency of 0.3 Hz is relevant only because it defines the Fourier fundamental frequency.

Figure 17 is identical to Fig. 16 except that the stimulus frequency is 3 Hz. One distinction between the low- and high-frequency results can be seen by comparing the surface plots for $P_1$ (Figs. 16C vs. 17C). In the 0.3-Hz case, the responses are always phase advanced. However, in the 3-Hz case, both phase advance and phase lag are observed depending on $I_0$ and $I_1$ (as in Fig. 14A). There is also a significant region of the response surface in which $\Gamma$ at 3 Hz is elevated compared with 0.3 Hz (Figs. 16D vs. 17D). This reflects phase locking that is more prominent at the higher frequency.

**DISCUSSION**

Our results provide further support for the functional differentiation between burst and tonic response modes in thalamic relay neurons. From intracellular recordings of relay neurons in the cat’s lateral geniculate nucleus in a thalamic slice preparation, we have used Fourier analysis to quantify the response properties of relay neurons in different discharge modes. We concentrated on the temporal tuning of four relay cell response measures: $F_0$, $F_1$, $P_1$, and $\Gamma$. We found that the temporal tuning of relay neurons in cat is highly dependent on response mode (i.e., burst vs. tonic). Also these differences with respect to response mode were well captured using a relatively minimal IFB neuron model. Because the behavior of this simple neuronal model closely resembles our experimental data, it should prove useful for predicting population activity.
Responsiveness

The responsiveness of relay neurons to temporal frequency was markedly different between burst and tonic firing whether we measured the mean response ($F_0$) or the fundamental, modulated response component ($F_1$). In terms of temporal tuning, burst firing was low-pass for higher values of stimulus intensity ($I_1$) but markedly band-pass for lower values of $I_1$ (Figs. 5 and 8, A and B). In contrast to burst mode, the responses in tonic mode was broadband for the stimulus frequencies and intensity levels tested (see Figs. 5 and 9, A and B). Thus the tonic responses to very low frequencies were relatively robust. Our results for burst firing are consistent with

**FIG. 15.** A–E, left: $F_0$ (open circles) and $F_1$ (filled circles) as a function of $I_1$ for 2-Hz IFB model responses. $I_0$ is depolarizing in A and becomes hyperpolarizing in E. Arrows labeled $V_{ss}$ indicate membrane potential that would be achieved due to the DC bias in the applied current (in the absence of spiking). Middle and right (labeled $I_1$ low and $I_1$ high): voltage time courses for $I_1$ values of 0.33 and 1.67 $\mu$A/cm$^2$, respectively. Dotted lines, tonic spike threshold, $V_u$; dashed lines, burst threshold, $V_h$. For A–E, respectively, $I_0$ in $\mu$A/cm$^2$: 2.0, 1.0, 0.58, 0.15, −0.43.
in vivo temporal tuning measurements derived from extracellular recordings in anesthetized cats (Mukherjee and Kaplan 1995), where retinogeniculate transmission during burst responses to visual stimulation were described having a temporal band-pass characteristic and during tonic firing were tuned more broadly. Also although responses in burst mode rarely were observed at 10 Hz regardless of $I_1$, neuronal responses in tonic mode routinely were observed at 30 and 100 Hz. This is qualitatively consistent with an in vitro study of guinea pig thalamic neurons (McCormick and Fezer 1990) that reported a high-frequency cutoff for repetitive burst responses near 10 Hz. The higher value obtained in this case may reflect the different stimuli used, i.e., square pulses versus sinusoids.

Another feature of the high-frequency cutoff in burst mode that we observed was the attenuation rather than an abrupt cutoff of the $F_0$ and $F_1$ values at 3 Hz measured as spikes/cycle (see Fig. 8B). This could arise from two different properties that we observed in the experimental results. First, in about half the neurons, we observed subharmonic responses at 3 Hz and this would result in a decrease in spikes/cycle when averaged over a number of trials. Second, although individual bursts at lower frequencies (<3 Hz) evoked essentially the same number of action potentials, neurons that did not respond with subharmonic bursts at 3 Hz often exhibited response attenuation via a decrease in the number of spikes/burst (Fig. 12D). Although the IFB model is too idealized to account for subharmonic burst responses, it does reproduce this reduction of spikes at higher frequencies (Fig. 12D). Previous computational work also has indicated that, during repetitive hyperpolarizing current pulses, $I_T$ gradually decreases at higher fre-
properties (Wang et al. 1991) and thus likely would evoke fewer action potentials.

Burst firing in response to sinusoidal current injection thus is tuned to roughly 0.3–3 Hz, particularly for lower amplitudes of activation. The failure of burst responses to higher frequencies is due to the low-pass filter properties of the membrane that lead to attenuated voltage responses at high frequencies of injected current (this also applies to tonic responses) and to the time constant for deactivation of $I_T$, which imposes a relative refractory period for the low-threshold Ca$^{2+}$ spike, thereby limiting the high-frequency response. The IFB model responding in burst mode reproduces the high-frequency attenuation of $F_0$ and $F_1$ (Figs. 10, A and B, and 12D). However, it should be noted that this success is due in part to the fact that these response measures do not distinguish between subharmonic burst responses and reduction of spike/burst at higher frequencies.

The attenuation of burst responses observed at lower frequencies with moderate $I_T$ values is a result of a fairly constant number of action potentials per burst in conjunction with one burst per stimulus cycle. When the response is measured in spikes/second, this results in an apparent attenuation at lower frequencies. However, the lowest $I_T$ values giving rise to a burst response at 1–3 Hz resulted in no response at 0.1 or 0.3 Hz in 6 of 11 cells tested (see Fig. 5). The origin of the low-frequency cutoff observed in some relay neurons can be understood by considering models that are biophysically more detailed than the IFB model. Such models can be tuned to conform to our experimental observation with one burst per stimulus cycle. When the response is distributed around the peak of the sinusoidal stimulus. The phase of response measures do not distinguish between subharmonic burst responses and reduction of spike/burst at higher frequencies.

Our experimental observation of band-pass relay neuron responses (in units of spikes/cycle) may be of particular interest for the following reason. Fast EPSPs, such as those activated via ionotropic receptors (Conn and Pin 1997; Pin and Duvoisin 1995; Recasens and Vignes 1995), create a large enough $dV/dt$ to activate $I_T$ reliably. Much slower EPSPs, such as those activated via metabotropic receptors (Conn and Pin 1997; Pin and Duvoisin 1995; Recasens and Vignes 1995), might be so slow that the cell may depolarize without activating $I_T$. This also raises the possibility that EPSPs activated by metabotropic receptors could bring the cell from a hyperpolarized, burst-capable voltage regime to tonic firing mode by moving $I_T$ from the deinactivated state to inactivated without ever activating $I_T$. If this metabotropic-based depolarization was strong enough, tonic firing would be evoked because there is no low-frequency attenuation for responses in tonic mode. We know that retinal inputs activate only ionotropic glutamate receptors on relay cells (Godwin et al. 1996; McCormick and von Krosigk 1992) and are thus likely to activate $I_T$ reliably when the relay cell is in burst mode. Both cortical and brain stem modulatory inputs activate ionotropic and metabotropic receptors (Godwin et al. 1996; McCormick and von Krosigk 1992; reviewed in Sherman and Guillery 1996), which raises the intriguing possibility that activation of metabotropic receptors via these inputs can change the relay cell’s firing mode without activating the cell.

**Phase of response**

During burst firing, both the experimental data and the IFB model show an overall phase advance that lags slightly with increasing temporal frequency (Figs. 8B and 10B). Because burst responses tend to be focused in phase, the quantitative value of $P_1$ is largely determined by the phase of burst onset. The phase advance exhibited by the IFB model at 0.1 Hz reflects the fact that $V$ passes over the burst threshold, $V_T$, during the upswing of the sinusoidal current injection; however, the quantitative value ($P_1 = 0.2$ cycles) is sensitive to both $I_T$ and $I_{N0}$ (e.g., the burst occurs earlier and is consequently more phase advanced with greater $I_{N0}$). The decrease in phase advance with increasing frequency reflects the passive membrane properties in the subthreshold regime ($V < V_N$), that is, $V$ can be delayed $\leq 90^\circ$ with respect to the applied current when the IFB model is stimulated at higher frequencies (see Eqs. A2 and A3 in APPENDIX).

Tonic firing is different because there is less of a phase advance in the experimental data with a slight lag with increasing frequency and virtually no advance or change with frequency in the model. The reduced advance compared with burst firing can be explained by the fact that tonic firing provides a sustained response that will more symmetrically distribute around the peak of the sinusoidal stimulus. The subtle difference between the phase advance seen in tonic firing experimentally versus none in the model can be explained by the modest spike frequency adaptation (Vergara et al. 1998; Wang 1998) seen in geniculate relay cells (Smith et al. 1999). Spike frequency adaptation means that the response to a sustained input gradually reduces with time, and the result with a sinusoidal stimulus is that the responses will be slightly stronger before the peak of the sinusoidal stimulus than after. This, in turn, is reflected in a slight phase advance. Because the IFB model does not have spike frequency adaptation, it does not show this phase advance for tonic firing.

**Linear summation of responses**

The index of nonlinearity also differed greatly between the two firing modes, at least for low stimulus frequencies (0.1–1 Hz). Here tonic responses showed excellent linear summation, whereas burst firing did not, and this difference was captured effectively by the IFB model (Figs. 8D–11D). Our analysis indicates that this difference in linearity at lower temporal frequencies is largely due to the greater rectification of burst than of tonic responses. With increasing frequency of stimulation, the nonlinearity index during tonic mode became very similar to burst mode. However, here our analysis indicates that this increase in nonlinearity for tonic firing is a result not of increased rectification of the response but rather of phase locking of the response to the stimulus. This is curious because analogous measures of linearity of responses of geniculate cells to visual stimuli recorded in vivo indicate that tonic responses are reliably more linear than are burst responses at temporal frequencies that includes the range tested here (Guido et al. 1996).
Our IFB model suggests an answer to this conundrum: when sufficient current noise is added to the neuron model, the phase locking seen for ≥3 Hz is largely eliminated, and tonic responses remain much more linear than burst responses. Similar effects have been observed in vivo during current injections of regular-spiking cells of the visual cortex (Carandini et al. 1996) Because there is clearly more noise, such as synaptic noise, during in vivo recording than the sort of in vitro recording represented in Fig. 9D, we would expect a reduction in phase locking during tonic firing in vivo. However, it is interesting in this context to note that Reich et al. (1997) do find evidence for some phase-locking in responses to drifting sinusoidal gratings during in vivo recording in the cat lateral geniculate nucleus, but the phase locking was seen only to the highest contrast gratings used.

**IFB model**

An integral part of this work is the construction and tuning of a minimal model that quantitatively reproduces salient features of the thalamic relay cell firing patterns, including the burst/tonic differences discussed in the preceding text. The conceptual novelty of the IFB model is the compact account of both burst and tonic response properties of relay cells in terms of two thresholds, \( V_h \) and \( V_m \), responsible for the activation of burst and tonic spiking, respectively. When responding in tonic mode, the IFB model is essentially a classical leaky integrate-and-fire neuron model. When responding in burst mode, the dynamics of a single slow variable, \( h \), characterize the inactivation and deactivation of \( I_T \) that subserves burst firing. Unlike more complicated thalamic relay cell models, the IFB model is highly constrained to fit the Fourier analysis of experimental responses presented here. It is remarkable that most of the Fourier analysis of experimental data is captured by the dual threshold character of the IFB model, and particularly so is the ability to quantitatively fit responses of cat geniculate relay cells with such a minimal characterization of \( I_T \). Our observation that low-frequency injected current often fails to activate \( I_T \) even though higher-frequency stimulation is successful as well as the observation that many neurons do not repetitively burst for any range of injected current, together suggest a relatively small, if any, window current in these neurons (Coulter et al. 1989). However, we have not explicitly probed for membrane potential bistability or other phenomenon that might be induced by a window current (Williams et al. 1997). Our success with the Fourier analysis using only a caricature of \( I_T \) is presumably due in part to the use of a (relatively simple) sinusoidal stimulus and our focus on neuronal responses at the (relatively coarse) level of PSTHs. Thus it will be important to explore the generalizability of the model by comparing experimental and theoretical responses to more complex stimuli. Similarly, although here we found it unnecessary to include in the IFB model the hyperpolarization-activated cation conductance, \( I_n \), in other contexts this conductance is known to play a significant role (McCormick and Pape 1990a).

Although the IFB model reproduces subharmonic tonic responses of relay neurons, it does not reproduce the subharmonic bursts occasionally observed experimentally. An important question is what aspect of the simplicity of the IFB model is responsible for this limitation. When relay cells are responding with subharmonic bursts, often a perfunctory low-threshold spike occurs on the crest of the cycle directly preceding a robust burst. Thus the relay cell appears to accumulate deactivation of \( I_T \) (i.e., \( h \) gradually increases) during missed cycles (i.e., those in which bursts responses do not occur). This suggests that the IFB model does not reproduce subharmonics in burst mode because the simplified kinetics of \( I_T \) used do not allow the gradual increase in \( h \).

To test this idea, we looked for subharmonic responses in a mathematical model of the low-threshold spike that uses continuous activation and inactivation curves for \( I_T \) that are biophysically more realistic than the discontinuous curves used in the IFB model (Wang et al. 1991). In agreement with our experimental observations, we chose parameters for this model such that repetitive low-threshold spikes (superharmonics) were not expressed for any range of applied current. In a narrow frequency range, this low-threshold spike model exhibited subharmonic responses similar to those of the relay neurons presented here. In addition, in the low-threshold spike model, deactivation \( (h) \) does indeed accumulate during missed cycles. Thus the inability of the IFB model to reproduce the subharmonic burst responses observed occasionally in experiments may be due to its minimal characterization of \( I_T \). On the other hand, reproducing subharmonic burst responses with a mathematical model that lacks \( I_h \) does not eliminate the possibility that \( I_h \) contributes to the experimentally observed responses. Indeed, during periodic square pulse stimulation, relay neuron models that include both \( I_T \) and \( I_h \) can produce subharmonic bursts that appear to originate from the temporal integration of hyperpolarization by \( I_h \) (Wang 1994).

**Conclusions**

There are two broad conclusions we can draw from these studies. First, on the experimental side, we have used Fourier techniques to characterize the input/output properties of geniculate relay cells in tonic and burst firing mode. Among the findings was evidence that tonic firing showed low-pass and sometimes broadband temporal tuning, while burst firing was band-pass, peaking near 3 Hz and often not responding to stimuli administered at lower frequencies (e.g., 0.1 Hz). These data are consistent with in vivo observations based on receptive field properties (Mukherjee and Kaplan 1995) and suggest that burst firing is not very responsive to steady-state properties of the visual world but rather require temporal change. We also found that, for frequencies <3 Hz, tonic responses were more linear than burst responses, whereas phase-locking of tonic responses ≥3 Hz increased the nonlinearity of tonic responses to levels near those seen during burst firing. Studies of receptive field properties in vivo find, on the one hand, that tonic firing is generally more linear than is burst firing for geniculate cells (Guido et al. 1992, 1995; Mukherjee and Kaplan 1995) and, on the other hand, that phase locking can occur for very high contrast stimuli (Reich et al. 1997). However, the relationships among phase locking, linearity, temporal frequency, and burst versus tonic firing mode in these in vivo studies has not been exhaustively explored. Nonetheless it seems clear that many of the receptive field properties noted above are at
least partly due to the membrane properties of the geniculate relay cells themselves, particularly with respect to \( I_T \).

Second, on the theoretical side, we have been able to reproduce the salient features of relay neuron responses to sinusoidal input with a minimal IFB neuron model. The computational simplicity of this model makes it a good starting point for network simulations of retinogeniculate transmission that include important aspects of thalamic circuitry (e.g., GABAergic inhibition from local interneurons and neurons of the thalamic reticular nucleus). One of the keys to understanding thalamic relay function will be to understand how thalamic circuitry and corticogeniculate feedback controls the burst and tonic firing modes of geniculate relay neurons.

**Appendix**

**Current-frequency relation for the IFB model in tonic mode**

When responding in tonic mode, the IFB model is essentially a classical leaky integrate-and-fire neuron model (Knight 1972). In the absence of bursting, \( h \) eventually decays to zero and as such the membrane response is not oscillatory. Thus, for \( h \), the low-threshold \( I_T \) current, \( I_T \), will not contribute to the dynamics of the membrane potential, \( V \), which are effectively given by

\[
C \frac{dV}{dt} = I_o + I_L \cos(2\pi f t) - g_L(V - V_L)
\]

(A1)

This equation differs from Eq. 3 presented in Methods only because \( I_T \) has been set to zero and the applied \( I_{app} \) and leakage \( I_L \) currents have been made explicit. As before, the membrane potential, \( V \), is reset whenever the membrane potential reaches the firing threshold, \( V_g \), such that \( V(t) = V_g \Rightarrow V(t) = V_{reset} \).

Until \( V \) achieves the threshold for firing an action potential, \( V_g \), the membrane potential of the model neuron is given by Eq. A1, solutions of which take the form

\[
V(t) = W(t) + [V(t_0) - W(t_0)] \exp[-(t - t_0)/\tau]
\]

(A2)

where \( t \) and \( t_0 \) are the present and initial time, respectively, \( t \) is the membrane time constant, given by \( \tau = C/g_L \), and \( W(t) \) is given by

\[
W(t) = V_L + I_o/g_L + \frac{I_d/g_L}{1 + (2\pi f t)^2} \left[ \cos(2\pi f t) + 2\pi f t \sin(2\pi f t) \right]
\]

(A3)

The current-frequency relation for a classical leaky integrate-and-fire neuron model can be derived from Eqs. A2 and A3. Setting \( I_0 = 0 \), the expression for \( W(t) \) simplifies to \( W(t) = V_L + I_d/g_L \), which is no longer a function of \( t \). To calculate the interspike interval, \( T \), for a given value of the DC-applied current, \( I_0 \), let \( V(0) = V_{reset} \) and solve for the first value of \( T \) such that \( V(T + t_0) = V_g \).

This gives

\[
V_g = V_L + I_d/g_L + [V_{reset} - V_L - I_d/g_L] \exp[-T/\tau]
\]

(A4)

which can be rearranged to give

\[
T = \tau \ln \left( \frac{I_d/g_L + V_L - V_{reset}}{I_d/g_L + V_L - V_g} \right)
\]

for \( I_d > g_L(V_L - V_L) \)

(A5)

where the condition on this expression indicates that the applied DC current, \( I_0 \), has to greater than the rheobase of the neuron for tonic spiking to occur.

**Estimates of cutoff frequency in tonic and burst mode**

Until either the tonic or burst threshold is crossed, \( V \) will be given by Eq. A2. If neither threshold is crossed for sufficiently long time, \( V \) will be given by \( V(t) = W(t) \) with \( W(t) \) as in Eq. A3. This solution represents the membrane potential oscillating under the influence of the sinusoidal applied current. Such responses were seen both experimentally and in simulations (Fig. 3, A and B, bottom). In the IFB model calculation in Fig. 3B, left, \( V \) is oscillating between the burst and tonic thresholds (i.e., in the gap region), whereas in Fig. 3B, right, \( V \) is oscillating below the burst threshold.

When the model is in the gap region, \( V_h < V < V_g \). Because \( V(t) = W(t) \) and \( W(t) \) is dependent on frequency, it is possible to ask how much the frequency would have to be lowered in order for a tonic or a burst response to be evoked by the model. This certainly will not occur unless the maximum membrane potential exceeds \( V_g \) (evoking a tonic spike) or the minimum membrane potential passes below \( V_h \) (required for a burst response). Focusing on the first possibility, we can find the maximum of \( V(t) = W(t) \) by differentiating Eq. A3 to give

\[
W'(t) = \frac{I_d/g_L}{1 + (2\pi f t)^2} (-2\pi f \sin(2\pi f t) + (2\pi f)^2 \tau \cos(2\pi f t))
\]

(A6)

Solving for \( t_{max} \) that satisfies \( W'(t_{max}) = 0 \) we find

\[
t_{max} = \frac{1}{2\pi f} \arctan(\frac{2\pi f}{\tau})
\]

(A7)

where \( 0 < 2\pi f \tau < \infty \) and \( 0 < \arctan(2\pi f/\tau) < \pi/2 \). It can be shown that \( W'_{max} < 0 \) confirming that this is a maximum. Substituting \( t_{max} \) into Eq. A3 we find that

\[
V_{max}(f, I_0, I_d) = V_L + I_d/g_L + I_d G(f)/g_L
\]

(A8)

where

\[
G(f) = \frac{1}{1 + (2\pi f/\tau)^2} \left( \cos(\arctan(2\pi f/\tau)) + 2\pi f/\tau \sin(\arctan(2\pi f/\tau)) \right)
\]

(A9)

and Eq. A8 has been written to emphasize that \( V_{max} \) is dependent on stimulus parameters as well as cellular parameters. In particular, note that \( G(f) \) is a monotonically decreasing function of the stimulus frequency, \( f \). Thus we can imagine lowering the stimulus frequency (thereby increasing \( V_{max} \)) until \( V_{max} \) achieves the tonic firing threshold, \( V_h \), which results in an implicit expression that gives an estimate of the cutoff frequency, \( f_{cutoff} \),

\[
\frac{V_h - V_L - I_d/g_L}{I_d/g_L} = G(f_{cutoff})
\]

(A10)

where the numerator of the left-hand side of the equality is positive (by our assumption that the membrane potential was originally oscillating between the burst and tonic threshold), and \( f_{cutoff} \) is only defined if \( G(f) > V_h - V_L - I_d/g_L \). From Eq. A10 one can see that \( f_{cutoff} \) is a function of stimulus parameters \( I_0 \) and \( I_d \) as well as cellular parameters; consequently, the experimentally observed values of \( f_{cutoff} \) should not be taken to reflect an intrinsic limit on a relay neuron’s firing frequency (e.g., the cutoff frequency in tonic mode for the neuron presented in Fig. 3A is only 3 Hz). Because \( G(f_{cutoff}) \) is a monotonically decreasing function, \( Eq. A10 \) implies that increasing \( I_d \) results in increasing the cutoff frequency in tonic mode. Also note that the cutoff frequency is dependent on the DC-applied current, \( I_0 \).

As a final point, notice that if we know the stimulus frequency, \( f \), we can solve for the values of \( I_0 \) and \( I_d \) necessary to elicit a bursts response. We arrive at an expression similar to \( Eq. A10 \)

\[
\frac{V_h - V_L - I_d/g_L}{I_d/g_L} = G(f)
\]

(A11)

that can be rearranged in the following manner
\[ I_{\text{current}} = - \frac{F_{\text{d}}(V_s - V_c)}{G(f)} + g(V_s - V_c) \]  

(A12)

This expression shows the tradeoff between \( F_{\text{d}} \) and \( I_{\text{current}} \) that can occur while still allowing \( V \) to achieve the threshold for firing a burst, \( V_c \). The line defined by this expression is negatively sloped and corresponds to the leftmost region of nonzero response that runs from near \( F_{\text{d}} = 0 \, \mu A/cm^2 \) and \( I_c = 0 \, \mu A/cm^2 \) to respective values of \(-4\) and \(3\) (see plot of \( I \) and \( P_I \) in Fig. 16, C and D).

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