Synaptic Efficacy and the Transmission of Complex Firing Patterns Between Neurons

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Faure, Philippe, Daniel Kaplan, and Henri Korn. Synaptic efficacy and the transmission of complex firing patterns between neurons. J Neurophysiol 84: 3010–3025, 2000. In central neurons, the summation of inputs from presynaptic cells combined with the unreliability of synaptic transmission produces incessant variations of the membrane potential termed synaptic noise (SN). These fluctuations, which depend on both the unpredictable timing of afferent activities and quantal variations of postsynaptic potentials, have defied conventional analysis. We show here that, when applied to SN recorded from the Mauthner (M) cell of teleosts, a simple method of nonlinear analysis reveals previously undetected features of this signal including hidden periodic components. The phase relationship between these components is compatible with the notion that the temporal organization of events comprising this noise is deterministic rather than random and that it is generated by presynaptic interneurons behaving as coupled periodic oscillators. Furthermore a model of the presynaptic network shows how SN is shaped both by activities in incoming inputs and by the distribution of their synaptic weights expressed as mean quantal contents of the activated synapses. In confirmation we found experimentally that long-term tetanic potentiation (LTP), which selectively increases some of these synaptic weights, permits oscillating temporal patterns to be transmitted more effectively to the postsynaptic cell. Thus the probabilistic nature of transmitter release, which governs the strength of synapses, may be critical for the transfer of complex timing information within neuronal assemblies.

INTRODUCTION

The nature of the neural code has led to much speculation (for a review, see Buzsaki et al. 1994; Eggermont 1998; Fuji et al. 1996). For example it has been proposed (Hebb 1949; Hopfield 1995; Perkel and Bullock 1968; Von der Malsburg 1981) that the coding of information in the CNS emerges from different firing patterns. Such codes may include the rate of action potentials (Georgopoulos et al. 1986; Shadlen and Newsome 1998), well-defined synchronous activities of the gamma type (40 Hz) particularly during binding (Singer 1993), and more complex temporal organization of firing in large networks (Nicollelis et al. 1995; Riehle et al. 1997).

Relevant to the present study, it has been suggested that chaos, found in several areas of the CNS (Pei and Moss 1996; Schaff et al. 1994), may also contribute to the neuronal code (Skarda and Freeman 1987, 1990; So et al. 1998; van Vreeswijk and Sompolinski 1996). But the validation of this hypothesis requires a demonstration that deterministic patterns can be, and are effectively, transmitted along neuronal chains.

This issue faces numerous difficulties, particularly in in vivo preparations, due to the variability of the ongoing activity in neurons called synaptic noise (SN) (Brock et al. 1952). This noise has been first attributed to a “random synaptic bombardment” of the recorded cells. Except for an early claim (Calvin and Stevens 1967) and some recent reports (Arieli et al. 1995 1996), the view according to which SN degrades neuronal functions has remained prevalent over the years (for a review see Ferster 1996). More important, this process has been commonly assumed to be stochastic (Calvin and Stevens 1967; Shadlen and Newsome 1998; Softky and Koch 1993), and it has been most often modeled as such (Mainen and Sejnowski 1995; Stevens and Zador 1998). Therefore recent studies on this intriguing phenomenon have mostly concentrated on whether or not, and in which conditions, such a Poisson process contributes to the variability of neuronal firing (Shadlen and Newsome 1994, 1995; Softky 1995). However, the renewed interest in SN leaves open the question of whether specific information about the state of firing of the presynaptic networks can be extracted from SN despite its random time appearance. The variability in both the amplitude and time of occurrence of the synaptic responses that build up SN precludes the sole use of conventional methods in solving this problem.

In this paper, we analyze the fine structure of SN in terms of timing and amplitude. SN was recorded intracellularly from the Mauthner (M) cell of teleosts, the command neuron of the adversive reaction to external stimuli (Zottoli 1977). Specifically we want to understand how SN reflects the state of the presynaptic networks and how synaptic junctions are involved in the transmission of this information. For this purpose, we examined physiological recordings using analytical tools based on nonlinear dynamics (for a review, see Abarbanel 1995; Schreiber 1999) already successfully applied to decipher the complexity of other neuronal systems (Guckenheimer and Rotar 1997; Pei and Moss 1996; Schaff et al. 1994). Our results indicate that, surprisingly, the fluctuating properties of synapses govern the degree to which complex activities in presynaptic networks are recapitulated postsynaptically and that this process is facilitated by a classical paradigm of learning.
Part of this work has been presented in an abstract form (Faure and Korn 1998a).

METHODS

Electrophysiological recordings

In the M cell of teleosts, the command neuron of the escape reaction from aversive stimuli, SN is inhibitory and is generated by two groups of glycinergic interneurons (Fig. 1A) one of which is driven by auditory inputs (Faber and Korn 1978). This activity was recorded in vivo in a quiet auditory environment with KCl-filled microelectrodes in the M-cell lateral dendrite of anesthetized adult goldfish (Carassius auratus, n = 14) and zebrafish (Brachydanio rerio, n = 16) as described in Faure and Korn (1997). Because the inhibitory postsynaptic potential (IPSP) in the M cell is hard to detect as a potential change (Furshpan and Furukawa 1962), Cl− was iontophoretically injected through the recording microelectrode until large and stable full-sized depolarizing collateral IPSPs evoked by antidromic activation of the M axon (Faber and Korn 1982) were recorded. Thus the IPSPs comprising SN also appeared as depolarizing potentials (Fig. 1B, top). Recordings were digitized at 12 kHz and filtered at 3 kHz with a low-pass Bessel filter.

Data analysis

An approximation of the time derivative of the recorded signal was calculated using first differences. The peak amplitudes and times of occurrence of the resulting upstroke “spikes” constitute the signal that was subjected to further analysis. These spikes could be easily resolved against the background noise. The spike train was sieved by ignoring all events whose amplitudes fell below a specified threshold, θ. Thus the spike sequence submitted for further analysis depends on both the signal itself and the imposed threshold.

The derived trains were examined with several techniques. Interspike event histograms were constructed using a kernel-based density estimator (Parzen 1962). The power spectrum of the spike point process was estimated using the discrete-time Fourier transform of the 12 kHz signal set equal to 1 at the time of events and 0 elsewhere.

Nonlinear structures in the spike trains were examined graphically using return maps, also referred to as Poincaré maps (PMs) (Faure and Korn 1997, 1998b; Garfinkel et al. 1992). The PMs were constructed by scatter plotting each interval between two successive events \( I(n+1) \) versus the previous one \( I(n) \). Note that in all maps presented in this report, each point corresponds to consecutive pairs of intervals \( (I_1, I_2) \) among three IPSPs. That is for the first data point of the illustrated series, \( I(n) = I_1 \) and \( I(n+1) = I_2 \), whereas for the second one, \( I(n) = I_2 \), \( I(n+1) = I_3 \), and so on.

Quantitative measurements of possible nonlinear determinism, or

FIG. 1. Experimental network and illustration of the threshold method. A: Mauthner (M) cell’s inhibitory pathways with commissural interneurons (Comm.) activated via VIII nerve fibers and the recurrent collateral (Coll.) pathway. + and −, excitatory and inhibitory connections, respectively. B, top: sample of synaptic noise (SN) recorded at a fast sweep speed with consecutive inhibitory postsynaptic potentials (IPSPs) observed as depolarizing events due to the Cl− loading and indicated by dots [\( V(t) \)]. Bottom: corresponding time derivative \( [dV(t)/dt] \). Note that the upstroke spikes can be easily distinguished from the background instrumental noise delineated by ·····. C, 1 and 2: derivative of a segment of SN recorded at a slow sweep speed; spike events each corresponding to a differentiated IPSP were selected by a threshold \( \theta \) (−−). C1: intervals between each detected event (In) and the next one \( (I_n + 1) \) were successively measured (●, top line) for further analysis. C2: as the threshold was lowered, more events were incorporated in the time series (−−).
otherwise stated of the degree to which the studied signals can be distinguished from random processes, were made as explained in APPENDIX A.

Modeling

A mathematical representation of the presynaptic networks and their dynamics consisting of four coupled model interneurons was built following Hindmarsh and Rose (1984) and Rose and Hindmarsh (1985). In this physiologically relevant and widely used model (Abarbanel et al. 1996; Hansel and Sompolinsky 1992; Keener and Sneyd 1998), each neuron \( i = 1, 2, 3, 4 \) is characterized by three time-dependent variables: \( x_i \), the membrane potential; \( y_i \), a recovery variable; and \( z_i \), a slow adaptation current. Let

\[
\frac{dx_i}{dt} = y_i - x_i + 3x_i^2 - z_i + I_0 + I_i, \tag{1}
\]

\[
\frac{dy_i}{dt} = 1 - 5x_i^2 - y_i, \tag{2}
\]

\[
\frac{dz_i}{dt} = [S_i(x_i - C_i) - z_i] \tag{3}
\]

where \( I_i \) is the external current into cell \( i \), \( I_0 \) is the injected current, \( C_i \) the voltage threshold for spike generation, \( S_i \) is the scale of the influence of the membrane voltage on the slow dynamics, and \( r \) is the time scale of the slow adaptation current. Depending on the values of these parameters, the individual neurons may be in a steady state and generate a periodic low-frequency repetitive firing (LFRF), chaotic bursts, or a high-frequency firing. In our tests, a LFRF was obtained with \( I_0 = 3.281, r = 0.0021, C_i = -1.6, \) and \( S_i \) was in the range of \([2.9–3.3]\). For each neuron, \( S_i \) was chosen to set fire at one of the frequencies detected by our analysis of experimental data.

Synapses linking neurons were formalized as

\[
I_i = - \sum_{j=1}^{N} g_{ij}(t-t_j)(x_j - E_m) \tag{4}
\]

where \( I_i \) is the total synaptic input to neuron \( i \), \( N \) the total number of inhibitory synapses established on their target by the modeled neurons (see diagram of Fig. 7A), \( E_m \) is the reversal potential of the inhibitory current (here set to \(-3\)), and \( t_j \) is the arrival time of a presynaptic spike. The latter induced a conductance change \( g_{ij}(t) \) having a time to peak, \( t_{pp} = 1 \) ms and a decay time, \( \tau = 6 \) ms, corresponding to physiological values determined experimentally at M cell’s inhibitory glycinergic interneurons (Faber and Korn 1978, 1982; Korn and Faber 1976). \( g_{ij}(t) \) was modeled with an alpha function according to Wilson and Bower (1989). The equations were solved with time steps of 0.001 ms, using a fourth-order Runge Kutta scheme that ensures that all interesting variations in the dynamics are represented (Abarbanel et al. 1996). Solutions were desampled to produce time steps of 0.05 ms.

Simulations produced different network behaviors depending on the value of \( g_m = g_{ij}(t = t_{pp}) \). These were synchronized antiphase bursts of action potentials for strong coupling, i.e., \( g_m \geq 10 \), asynchronous and intermittent firing for intermediate coupling \( 3.5 \leq g_m < 10 \), and continuous firing of the neurons with slight fluctuations around a mean frequency for small coupling values such as \( g_m = 3.5 \).

Simulations for this paper corresponded to the third case.

Induction of LTP

LTP of M-cell inhibitory synapses was produced by trains of sounds delivered by a loudspeaker placed in the vicinity of the fish (50 ms, 500 Hz, and 75 dB) repeated every 4 s over 3 min (Oda et al. 1998). As in previous studies (Korn et al. 1992; Oda et al. 1995), the method for assessing the resulting increase of inhibitory synaptic strength was based on measurements of the reduction in the antidromic spike height due to the inhibitory shunt. Since this action potential propagates passively into the soma (Furshpan and Furukawa 1962), any conductance change can be calculated as \( r' = (V/V') - 1 \), where \( V \) and \( V' \) are spike amplitudes in the absence and presence of inhibition, respectively. This expression represents the ratio (or fractional conductance) \( G_{psp}/G_{an} \), the two terms being the inhibitory and resting conductances, respectively (Faber and Korn 1982).

RESULTS

In previous studies, we have shown that nonrandom patterns can be observed in SN (Faure and Korn 1997, 1998b), but their detailed organization has not been elucidated despite their striking appearance in the form of well-delineated triangles in PMs. Thus several questions have been addressed in the present study. The first was to identify the classes of dynamical systems and related synaptic events that can produce such geometric figures. The second was to determine what can be inferred about the functional organization of the presynaptic networks that generate the recorded SN. This led us to investigate how the firing patterns of these networks are transferred to postsynaptic cells. The overall conclusion of this investigation is that even though synaptic transmission is probabilistic, SN is a true signal that offers insight on the state of firing of the presynaptic networks.

Periodic components in SN

The recorded signal \( V(t) \) showed large oscillations made of successive inverted IPSPs with a magnitude up to several millivolts (Fig. 1B, top). The time derivative \( dV/dt \) of this signal provides an index of the onset and size of each event: the beginning of an IPSP was apparent as a rapid increase in slope reaching a maximum during the rising phase of the synaptic potential (Fig. 1B, bottom). The size of the resulting spike was proportional to that of its parent IPSP.

Figure 1C illustrates the basic method used for this study. Subsets of events were selected, according to their amplitudes, by a threshold \( \theta \) in such a manner that as this threshold was lowered an increasing number of events was included in the resulting time series. The use of multiple-threshold levels to produce corresponding trains of events derived from the same \( dV/dt \) recording (Fig. 2A, 1 and 2) allowed us to take into consideration the information contained in both their timing and relative amplitudes.

For example, the distribution of interspike intervals (ISI) obtained with the events which crossed the highest \( \theta = \theta_1 \) were multimodal (Fig. 2B1). This confirmed their regular periodicity (see also Faure and Korn 1997), which can also be evidenced with autocorrelations (Hatta and Korn 1999). But for lower \( \theta \), this periodicity was blurred, and time intervals possessed no obvious structure when the lower-amplitude IPSPs were taken into account (Fig. 2B, 2 and 3).

Power spectra of the spike point process showed additional organization of the trains. In the illustrated case, even at \( \theta_3 \) (Fig. 2C), there was a broad peak between 60 and 80 Hz. This result was interesting but difficult to interpret. The simplest explanation, that the events were approximately periodic with a period of roughly 12–16 ms, was clearly ruled out by the ISI histogram (Fig. 2B3), which shows that interspike intervals
periodic components were buried in SN. For this purpose, events associated with $f_p$ were subtracted from the time series isolated by $\theta_1$ (Fig. 3B1), and the maps were reconstructed with the remaining events. This procedure disclosed a distinct second signal-flag pattern, centered on a secondary frequency $f_s$. In this experiment, $f_s$ was equal to 68 Hz (Fig. 3B2), and overall we found $f_p < f_s$ in the 12 of 21 experiments where additional triangles could be revealed.

These results suggest that the IPSPs in SN are organized in a primary periodic train of high-amplitude and frequency ($f_p$) and in a second train of somewhat lower amplitude and frequency ($f_s$). As shown in the following text, other intervening and smaller events correspond to at least a third oscillator, ($f_t$).

**Interpretation of the PMs**

A simple theoretical analysis consistent with experimental data helps to interpret the signal-flag pattern. Consider a sequence of events (Fig. 4A) consisting of evenly spaced IPSPs (labeled P) having an amplitude greater than $\theta_1$ and period of $\pi_p$ and another sequence of smaller events (labeled S) having an amplitude near $\theta_2$ and period $\pi_s$, which are intermingled with the larger P components. The sequences that are used to construct the ISIs depend on the setting of the amplitude threshold. Overall, maps constructed with interwoven periodic S and P events and with some missing S will produce a PM with points scattered on the four lines shown in Fig. 4B.

Specifically, when the threshold is between $\theta_2$ and $\theta_3$, only the P events will be detected. The resulting sequence can be denoted $\text{PsPsPsPs}$, where capital letters stand for events that are above the threshold. At this level, the sequence of ISIs will be $\pi_p, \pi_p, \pi_p, \ldots$ and the PM will be a single dot at the position marked $\text{PsPsP}$. A small amount of random variations in $\pi_p$ will broaden this dot into the type of the circular cloud seen experimentally.

At a lower threshold, near $\theta_3$, some S events will be detected and the resulting sequence of events is $\text{PsPsPsS}$. The ISI comes from consecutive triples, e.g., PSP, SPS, or SPP. Pairs of intervals from PSP triples, e.g., $I_1, I_2$ and $I_1, I_3$, in Fig. 4A span the period $\pi_p$, and they appear in the PM along the diagonal line marked $\pi_p$. For SPS triples, the dot can be anywhere in the square bounded by $\pi_p$, but when the S-type events are periodic, SPS triples lie on the diagonal line corresponding to $\pi_s$. When S events fall below $\theta_3$, triples will be of the form $\text{PsPsS}$, $\text{SpsP}$, or $\text{PsPsP}$. These appear on the vertical or horizontal lines of the signal-flag pattern or at their intersections, respectively.

Finally, PMs constructed with thresholds below $\theta_3$ also exhibit points in the lower left triangle already evident in Fig. 3A, 2 and 3. These points correspond to the events denoted T that were also assumed to be part of a periodic sequence.

A careful examination of the experimental PMs confirmed this interpretation. For example, the principal and secondary periods $\pi_p = 16.25$ ms and $\pi_s = 14.4$ ms detected at the summit of the triangles in Fig. 3A2 and B2, were also apparent at the border of the highest density areas when the PMs were converted into density maps (Fig. 4C). Applying the same protocol to the triangle obtained after excluding P events disclosed a third period $\pi_T = 13.3$ ms at the lower edge of the signal-flag pattern (Fig. 4D). The values of these three periods
helped to make sense of some of the peaks revealed by the power spectrum of Fig. 2C (arrows).

Are the presynaptic oscillators coupled?

The experimental PMs, corroborated by the power spectra, indicated that activities in the presynaptic interneurons that generate SN are rhythmic. However the linear signal-flag structures in the experimental PMs were broader than those expected if the P and S events were exactly periodic (in which case they would lie exactly on the 4 lines of Fig. 4B). Two explanations were possible. One was that \( \pi_p, \pi_s, \) and \( \pi_f \) were independent of each other and varied randomly. The other was that, as in other neuronal systems (Keener and Glass 1984; Pei and Moss 1996), the parent oscillators of the P, S, and T events are coupled to one another, thus producing fluctuations in the periods.

An analysis of the time intervals between the IPSPs comprising SN was carried out to distinguish between three distinct alternative possibilities. First, if the two oscillators P and S are independent, S events will occur with equal probability at any position in PP intervals (Fig. 5A, top). Second, if the oscillators are phase locked, for example by a strong synaptic path with a fixed delay, PS intervals will remain constant (Fig. 5A, middle). Finally, in case of a functionally weak synaptic coupling, the oscillators exert complex influences on each other and depending on the previous timing of P and S events, the phase of a subsequent one will be advanced, retarded, or remain the same (Fig. 5A, bottom).

When PSPSP sequences corresponding to periods \( \pi_p \) and \( \pi_s \) were extracted from SN and ranked as in Fig. 5B1, we found that intervals \( P_1P_2 \) and \( S_1S_2 \) were strongly correlated and that S events were not homogeneously distributed between P ones as would be the case if the oscillators were independent. This phase relationship was quantified, as illustrated by the plot of Fig. 5B2 (black circles) in which the regression line shows a statistically significant (\( P < 0.001 \)) rejection of the null hypothesis that the SS and PS intervals were independent. Furthermore when the largest, though unclear (due to their timing) S events were incorporated in the analysis (Fig. 5B2, blue circles), the slope became even more
pronounced. Thus the plot of Fig. 5B2 can be taken as indicating that the S events did not occur during the first 20% of the PP cycle, SS intervals were dramatically prolonged when a S event occurred in the first half of the PP interval, and these intervals were shorter when the S events occurred later in the PP sequence. These three forms of mutual interaction are consistent with a weak coupling between oscillators.

This coupling was not strong enough to phase-lock the S and P events to a stable and constant interval, but it could produce short sequences that exhibited almost constant phase relationships between the P and S oscillators. Advance and retreat patterns (ARPs), similar to those seen previously (Faure and Korn 1997), were observed in the PMs (Fig. 6A). These stemmed from the slow drift in phase between S and P events. When the two oscillators were half a cycle out of phase they appeared as a fixed point (Fig. 6A, 2) but since they were not phase locked, this point was unstable and the next points labeled 3 and 4 in the PM diverged along a well-defined path resembling that of unstable periodic orbits (UPOs) often associated with chaos (So et al. 1998). Other types of period 1, 2, and 3 orbits were also found (Fig. 6B).

These different configurations correspond to the various sequences of time intervals between events outlined in Fig. 6C. They stress the diversity of temporal structures contained in SN. The period-1 orbits correspond to a single interval between successive events. Period-2 orbits correspond to two different alternating intervals, while period-3 ones include three distinct sequential repeated intervals and these iterations can be generalized to n. The construction of the ARPs is more complex: the successive intervals converge toward and tend to stabilize around a fixed one, but they rapidly escape following distinct paths as illustrated in Fig. 6A.

Finally it should be stated that the measures of determinism, i.e., the percentage of determinism (%det) and the $\mu(e)$ entropy (see details in APPENDIX A), were statistically significant in 19 of the 21 experiments when compared with surrogates, confirming the nonlinear properties of SN.

Complex patterns in presynaptic networks

Numerous reports have demonstrated that coupled neurons can behave as oscillators and generate a vast repertoire of

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**FIG. 4.** Construction of the different parts of the signal-flag pattern. A: the notation is explained with a schematic dV/dt time series comprising 3 classes of periodic IPSPs, labeled P, S, and T according to their parent frequency. Events below a given threshold of analysis are in lower case. B: interpretation of a theoretical map using the same notation as in A to designate the origin of the data points lying along the different arms of the triangle. Lines $\pi_p$ and $\pi_s$ mark the diagonals determined by the principal and secondary periods. Points below concern intercalated T’s. C: evidence that the principal ($\pi_p = 16.25$) and secondary ($\pi_s = 14.4$) periods fit the highest density of points on a PM constructed with a set of 6 successive time series corresponding to 10 s of recording each (same experimental data as for Fig. 3A2). The density was calculated by partitioning the space in $50 \times 50$ square areas (i.e., with a resolution of $0.42 \times 0.42$ ms) and by counting the number of points in each of these boxes. Areas in blue, green, red, and yellow indicate regions containing less than 4 points, between 4 and 8, 8 and 12, or more points, respectively. D: same protocol as for C applied to the experimental data of Fig. 3B2. Note that the return map is consistent with the 2-period theory, but shifted: the slow periodicity is now $\pi_q = 14.4$ ms and a faster period, $\pi_e = 13.3$ ms is unmasked. Here colors pertain to areas containing less than 2, between 2 and 4, 4 and 6, or more points, respectively.
dynamic responses, ranging from periodic to chaotic firing patterns (Abarbanel et al. 1996; Borisyuk et al. 1995; Hansel and Sompolinsky 1992; Rinzel et al. 1998). Since periodic events compatible with the involvement of coupled presynaptic “oscillators” were detected in SN, we assumed that the role of the M cell in the oscillations is simply a read-out function, and we investigated whether coupling between inhibitory interneurons terminating on the M cell can produce complex patterns similar to those observed in actual data. We found that a deterministic model, which does not involve random fluctuations, can reproduce all the major features of the signal-flag geometry, including the broadening of the diagonal PSP and SPS lines of Fig. 4.

Four interneurons were modeled as described in METHODS. They were linked by inhibitory synapses (Fig. 7A) generating IPSPs having a fixed latency and a constant amplitude to eliminate all sources of randomness. Values of this model’s parameters were set to obtain low-frequency periodic patterns of firing of the same order than those of the different classes of IPSPs revealed in PMs.

Figure 7B shows that each of the neurons produced trains of action potentials that were roughly periodic but fluctuated around a given mean period. Such deterministic fluctuations might account for the complexity of the experimental time series (see following text). However, the sum of the impulses produced by the four neurons took the undefined form of a random process and the corresponding return maps (Fig. 7C) were similar to those obtained by plotting events selected in actual data by a low threshold, as in Fig. 3A3.

Since most highly structured PMs were those constructed with intermediate thresholds θ2 and suggested the interplay of rhythms from two oscillators, we focused our attention on the behavior of two of the modeled neurons alone. For example, in the case of Fig. 8A, the two investigated cells fired with a mean frequency of 57 and 63 Hz, respectively. Yet, the intervals between the action potentials in the summed train were irreg-
ular with phase shifts analogous to those produced by weakly coupled oscillators.

Return maps constructed by plotting these intervals exhibited a well-delineated motif made of series of points distributed around $p_P$ and $p_S$ (Fig. 8B1). That is, they were distributed in the same region of the base of the signal-flag pattern obtained experimentally. One can also note in the PM a few distant points (crossed arrow) that mark the intervals between action potentials occurring before and after a pair of synchronous ones (Fig. 8A, crossed arrow). They are the precursors of the summit of a complete signal-flag pattern.

The scattering of points around $p_P$ and $p_S$ indicates that the weak coupling between the oscillators produced a deterministic dispersion of the time intervals between events (Fig. 8B1), although the simulations did not incorporate external sources of noise. These deterministic fluctuations produced ARPs that resembled those observed in the experimental data (not shown) and period-2 orbits (Fig. 8B2).

Role of synaptic properties in the transmission of presynaptic patterns

The various components of the signal-flag patterns were all evident in the same train because some types of events were sometimes above and sometimes below threshold, the fluctuations of their amplitudes allowing the signal-flags to convey information about the periodicities of each of the oscillators. In addition the distribution of amplitudes allowed for gradual transitions among the types of signal-flag patterns as the detection threshold was lowered, with, consequently, a reduction in the number of failures of detection. Therefore by examining the fraction of missed events at each level of $\theta$, we could approximate the extent of the overlap of some IPSPs produced.
by each oscillators (APPENDIX B 1). The amplitude fluctuations of these IPSPs were consistent with an involvement of synaptic junctions. We found that indeed, connecting the formal interneurons with terminal synapses that “released” transmitter in quanta according to principles established at chemical junctions allowed the model to reproduce the hierarchical features of the PMs.

Chemical transmission is governed by two parameters, \( n \) and \( p \), where \( n \) represents the population of basic “quantal units” \( q \) capable of responding to a nerve impulse, and \( p \) their average response probability (Del Castillo and Katz 1952). Then the product \( np \) is the so-called mean quantal content, which is equal to the average number of quanta released by a given junction during successive trials, while the product \( npq \), where \( q \) is the size of a quantum (set to 1 in our simulations), determines the synaptic strength. Since at central synapses transmitter release can follow a simple or a compound binomial statistics, where \( p \) is the same or is different for every site of release established by a neuron on its target (for a review, see Korn and Faber 1991; Redman 1990), we tested these two models in this study.

First, a simple binomial model, previously shown to account for the amplitude fluctuations of IPSPs evoked in the M cell by its presynaptic interneurons (Korn et al. 1982), was used with values for the terms \( n \) and \( p \) in the range of those derived in earlier experiments (Korn et al. 1986).

When the neurons had the same quantal content \( np \), all IPSPs fluctuated in the same range (Fig. 9A, 1 and 2) and the signal-flag pattern did not appear in the PM (Fig. 9A3). This result was easily explained by the fact that any threshold detected the same proportion of IPSPs regardless of their parent cell and despite small changes at the extremities of the inverse cumulative densities of the amplitude events (Fig. 9A2, 2). These slight discrepancies were due to small differences in the coefficient of variation (CV) of the amplitude distributions of the IPSPs produced by each cell. Similar conclusions were reached with other runs, whatever the values attributed to \( n \) and \( p \) of each interneuron, as long as the four modeled cells had an identical quantal content (a justification of this rule, which also pertains for compound binomial statistics, can be found in APPENDIX B 2).

When the model was modified to include different \( np \) products that distinguish presynaptic neurons from each other (Korn et al. 1986), the signal-flag pattern was restored in the maps and the ARPs reappeared, as well as period-2 orbits (not shown). This result is illustrated in Fig. 9B, 1–3, for which the “neurons” had a quantal content of 13.3, 6.2, 3.7, and 1.86, respectively. Again, this result can be easily explained. As
shown by Fig. 9B, 1 and 2, at intermediate values, \( \theta \) identified preferentially IPSPs produced by the first two cells. For example, when \( \theta \) was set to select the same proportion of events (i.e., \( \approx 42\% \)) as in Fig. 9A2, 100% of events from cell 1 and 60% from cell 2 were suprathreshold, against only 10 and 0% from cells 3 and 4.

Second, the model was implemented with terminals “releasing” according to nonuniform \( p' \)s. As illustrated in Fig. 10, the results were almost indistinguishable from those described in the preceding text. That is, a signal-flag pattern, ARPs and period-2 orbits became only apparent in PMs when the \( np \) products were different. This finding was not surprising since the distribution histograms of IPSPs modeled by a compound binomial are roughly similar to those from a simple binomial distribution but with a smaller CV than with a simple binomial statistics.

Finally it should be noted that in these simulations values of \( n \) and \( p \) were inversely correlated as experimentally observed at M cell’s inhibitory connections (Korn et al. 1986). However, this need not be the case (see APPENDIX B 2).

**Experimental validation**

To verify the involvement of synaptic efficacies, we conducted a series of experiments, taking advantage that the strength of the M cell’s inhibitory synapses are modified in vivo by LTP, a classical paradigm of learning that can be induced in teleosts by trains of sounds emitted in the vicinity of the fish. This form of LTP is due to an increase of the presynaptic parameter of release \( p \), while \( n \) and \( q \) remain unaffected (Oda et al. 1995). As in previous reports, the potentiation of the inhibitory synapses was quantified using a parameter \( r \), or fractional conductance, which defines the degree of the inhibitory shunt (see METHODS ).

As expected the amplitude of IPSPs comprising SN was increased during LTP (Fig. 11, A1 and B1). Furthermore return maps constructed with a high-threshold (\( \theta_2 \)) were markedly different before and after the conditioning sound trains (Fig. 11, A2 and B2). That is, LTP strengthened the two-oscillator triangular pattern in the PM, as further evidenced by comparing the PMs constructed with a lower threshold, \( \theta_2 \) (Fig. 11, B2 and C) and by the 8% increase of the \%det (Fig. 11D). On the other hand \( \pi_P \) and \( \pi_s \) remained the same (Fig. 12), suggesting a stability of the dynamics in the network.

In four experiments, \( r' \) was increased by 29 ± 8.1% (mean ± SE) after the learning protocol, indicating LTP of the commissural synapses (Korn et al. 1992), and the \%det was enhanced by 14.3 ± 2.1%. Adding three other experiments,
during which LTP was only assessed by comparing the amplitude of IPSPs (to avoid further modifications of SN by control sounds), this value became \(11.3 \pm 1.8\%\). In another cell, the sounds produced no LTP and the determinism was unchanged.

**DISCUSSION**

Taken together these results suggest that the information contained in SN permits the dynamics of the presynaptic networks to be reconstructed. This information is contained both in the intervals between IPSPs and in their amplitudes.

Our data can be generalized as follows. Several combinations of P and S events in triplets, some of which result from failures to detect events, are necessary to construct a complete signal-flag pattern. This prerequisite is guaranteed by the “separating power” of the largest \(np\) products at the terminals of neurons that generate the SN. On the other hand identified dynamical sequences (ARPs, period-\(n\) orbits and other signs of determinism) are found at the base of the signal-flag pattern, and the clarity with which they appear is related to the emergence of both P and S events in the time series. This second condition evidently imposes an irreducible separation between the \(np\) products (see APPENDIX B2) that allows the presynaptic dynamics to become predominant in the SN.

**Validity of the coupling hypothesis**

Two important issues had to be considered before accepting that the coupling between presynaptic interneurons accounts for the patterns observed in SN.

First, might we have incorrectly classified as P, S, or T activities? Our methodology for displaying the coupling between the P and S oscillators relies on the detection of events and their classification as P, S, and T. Thus any failure of detection or classification that is related to the timing of the spikes could artifactually influence the display of coupling. However, when constructing Fig. 5B2, we remained conservative and considered only events (shown in black) belonging to unambiguous PSPS quartets. P, S, and T events were selected using a criterion that combines amplitude with timing information. Specifically, P events were obvious and, despite possible confusions between S and T events due to overlap of their size, most S ones could be identified as such in Poincaré maps since they appeared on the diagonal line corresponding to the S period. Second, how reliably could we detect closely spaced events? Events closer than a given time separation were not distinguished in the time series. This issue was important because a sufficiently long lockout can produce the illusion of coupling. But this lockout was approximately 0.5 ms, and there are few points with short \(\phi\) in Fig. 5B2 simply because there were few PSPS quartets. Theoretical analysis shows that the 0.5 ms lockout is too short to produce artifacts mimicking the pattern in Fig. 5B2. In confirmation, simulations with uncoupled S and P oscillators show virtually no coupling even with lockouts as large as 3.5 ms. Two additional arguments reinforced the hypothesis of coupling. Data points in the PMs were not uniformly distributed as they would be if the variability of each frequency was random and, as demonstrated by combined electrophysiological and histological studies (Korn et al. 1990; Triller and Korn 1981), commissural inhibitory interneurons presynaptic to the M cell are linked by chemical inhibitory synapses.

Other simulations can produce triangular maps. Such is the
case for models based on one oscillator producing large events intercalated with smaller ones occurring at random (not shown) or on several independent and noisy oscillators, each making events of different amplitudes, and a fixed level of detection. However when examined in detail, such maps do not exhibit the fine temporal structure (periodic orbits, phase relationships) found in the SN recorded from the M cell.

Finally could nonlinearity in synaptic transmission and/or the responsiveness of the M cell to its inputs create the observed patterns? While this is conceivable, the most obvious sources of nonlinearity so far identified in the M cell system are unlikely to be involved. In particular the effects of presynaptic depression that reduces IPSP amplitudes at high rates of presynaptic firing are stabilized at frequencies more than 33 Hz (Korn et al. 1984). In confirmation no correlation was found between the amplitudes of the successive P events or between their amplitudes and timing. At the postsynaptic level, we investigated whether the nonlinear summation of potential change (Martin 1955) and, more importantly, the voltage dependence of the decay of IPSPs (Faber and Korn 1987) did affect the size of overlapping events. Such seemed not the case since the amplitude distribution of S events was the same during the first half of the PP cycle (which includes the falling phase of the P events) and during the second one.

Neuronal correlates of the presynaptic oscillators

Nonlinear deterministic components have been identified in SN (Faure and Korn 1997, 1998b), and it is well established that coupled oscillators can generate similar patterns, including chaotic ones (Glass and Mackey 1988). The frequencies of the oscillators unmasked in this study are consistent with those of the periodic components already noted in the M cell’s inhibitory SN (Faure and Korn 1997; Hatta and Korn 1999), which were in the range of the so-called gamma rhythm observed in higher vertebrates (Jefferys et al. 1996; Singer 1993).

Anatomical and physiological studies have shown that there are more than four interneurons in the M-cell presynaptic network with the estimates being in the range of at least 50 (Faber and Korn 1978; Korn and Faber 1990). Thus it is surprising that the contribution of a few oscillators can be distinguished or, in other terms, that the amplitude of the IPSPs produced by each of these oscillators seems to be ordered according to their origin. Calculations of the average size of the P and S IPSPs may help to address this issue. In the goldfish M-cell system, the size of an inhibitory
quantum is approximately 1% that of the full-sized collateral IPSP evoked by antidromic activation of the Mauthner cell axon (Korn et al. 1982). The mean unitary IPSPs produced by stimulations at 1 Hz of a single presynaptic interneuron comprise 5.8 quanta (Korn et al. 1986). Furthermore, their amplitudes decrease in a known way at increased stimulating frequencies due to presynaptic depression (Korn et al. 1984). Based on these values, estimations made in time series from three goldfish indicated that the P and S IPSPs were, respectively, six to seven and four to five times bigger than the average unitary response.

Several hypotheses that have not been tested in this study might explain this amplitude distribution. One is that these IPSPs are produced by the firing of a special set of cells, that of the rather exceptional “superinterneurons,” which evoke IPSPs of unusually high amplitudes (Korn et al. 1986). The other is that a number of presynaptic cells might fire simultaneously. Two signaling mechanisms already demonstrated in the organization of the M cell’s inhibitory network could be involved: one is the chemical coupling between inhibitory interneurons that has been shown to underlie synchronization in theoretical models (Jefferys et al. 1996; Traub et al. 1996; White et al. 1998; Whittington et al. 1995) and in experimental material (Bragin et al. 1995; Whittington et al. 1995). The second is that the somata of the commissural interneurons are linked in teleosts via gap junctions (Korn et al. 1977), which favor their simultaneous discharge including those of functionally related inhibitory cells (Gibson et al. 1999).

Arguments offered in APPENDIX B 3 indicate that our results do not depend on whether the quanta are issued by synchronously firing cells or are all from a single presynaptic neuron.

Synaptic properties and the transmission of deterministic patterns

Analysis of the PMs indicated that the signal-flag patterns were clearly delineated with maximum values of the measures of determinism [\(\%\text{det} \) and \(\mu(e)\)] and the largest number of ARPs, when 99.3 ± 0.32% \((n = 10)\), 49.7 ± 2.57% \((n = 10)\), and 12.1 ± 1.89% \((n = 10)\) of P, S, and T events were included in the maps, respectively. These values agree closely with those of the \(np\) products required for the construction of “meaningful” maps with models.

We observed in experimental data that the same sequence of event intervals can produce qualitatively different patterns depending on the level of the threshold used to window the sequence. Indeed, the probabilistic nature of transmitter release means that there is a graduated transition between the different patterns as \(\theta\) is varied. The notion that the probabilistic component \(p\) is critical here was confirmed by simulations in which the oscillators had different \(n\)’s but with \(p = 1\) at all terminals, reducing the variance to \(\sigma^2 = 0\). This resulted for any level of \(\theta\) in maps lacking signal-flag patterns. The nonzero variance that results from \(0 < p < 0.1\) guarantees that multiple components of the patterns shown in Fig. 4B can be present in the signal flags. The value of \(p\) sets the relative mixture of these patterns. Changing \(p\), as in LTP, modifies this mixture.

In this context, the probabilistic aspect of neurotransmission, whose function in neuronal communication has not been clarified, becomes an advantage rather than a limiting factor (Zador 1998) since it allows synaptic strengths to be adjusted in a manner that shapes the transmitted information without needing to modify the dynamics of the presynaptic network.

The emergence of the deterministic structures in the postsynaptic cell with multiple inputs is made possible by the nonuniform values of synaptic weights and the stochastic release of quanta. Furthermore the transmission of qualitatively different patterns in the M cell may allow encoding of a small repertoire of motor reactions as suggested by the results of LTP which is known to underlie modifications of the goldfish escape behavior (Oda et al. 1998).

APPENDIX A

A deterministic system is one whose complete behavior can be expressed with an infinite precision in a mathematical description (usually a differential equation). Then if the initial values of all
variables are known completely, the system’s entire future history is predictable and can be calculated exactly.

A chaotic system, which often appears at first sight to be random, is also deterministic, but it displays what is called sensitive dependence to initial conditions. That is, solutions obtained with two different starting points can be profoundly different: if the initial conditions are infinitesimally displaced from each other, then the solutions diverge exponentially. The extent to which they are unpredictable depends on this rate of divergence.

The dynamics of deterministic systems are different in principle from random processes where prediction is impossible except statistically.

While the presence or absence of a specific form of determinism was not the focus of this report (for this aspect of the work, see Faure and Korn 1997, 1998b), that of SN was quantified for two reasons. The first was to confirm that SN could be distinguished from a random process. The second was to estimate the modifications produced by the second was to estimate the modifications produced by so-called recurrent patterns, i.e., sequences which approximatively repeat themselves over time (hidden rhythms). They can be computed in recurrent plots (Eckmann et al. 1987), which are particularly well suited for studies of biological processes.

Let \( x(t) \) be the \( t \)-th point on the orbit describing a dynamical system in a \( d \)-dimensional space, for \( t = 1, \ldots, N \). The recurrence plot (RP) is an array of dots in a \( N \times N \) square, where a recurrent point is placed at the \( (i, j) \) coordinates whenever the embedded vectors \( (x_i, x_j) \), defined here by \( x_i = [x(t), x(t+1)] \) are within a predetermined cutoff distance \( \epsilon \). The organization of recurrent points into diagonal line segments of length \( L \) indicates the parts of the trajectories which are close (under resolution \( \epsilon \)) during \( L \) successive time steps.

The first parameter was the percentage of determinism \( \% \text{det} \) (Weber and Zbilut 1994), which is the number of dots included in diagonal line segments divided by the total number of dots in the RP. The second parameter was the slope, \( \mu(\epsilon) \), of the exponential decay of the histogram of the number of segments of length \( L \). As demonstrated elsewhere (Faure and Korn 1998b), at limits, this slope is an estimation of the Kolmogorov-Sinai entropy.

To confirm the nonlinear properties of the PMs, we constructed surrogate data. A surrogate is an artificial set constructed from the original data with constrained statistical properties that depend on the null hypothesis being tested. The statistical significance of the two parameters was examined with the null hypothesis that all forms of determinism found in SN were brought about by the linear properties (amplitude and frequency distributions) of the signal. For this purpose, surrogates of the time derivative of the membrane potential and of the raw spike trains, which matched both the amplitudes of the signals and their power spectrum, were constructed using two distinct methods despite their possible limitations when applied to time interval series (Schreiber and Schmitz 2000). These were the amplitude-adjusted method (Rapp et al. 1993; Theiler et al. 1992) and the iterative surrogate technique (Schreiber and Schmitz 1996), which involves an iterative refinement of the latter.

**Appendix B**

This appendix is concerned with the synaptic properties that can lead to the the signal-flag patterns. We treat the amplitude of each event (i.e., of an IPSP in SN) as random and seek to characterize the amplitude distributions of the P, S, and T events.

**Appendix B1: percentages of detection**

We need some statement about amplitude distributions of each class of IPSPs to express their degree of overlap. Since the release mechanism may obey simple or compound binomial or possibly other statistics (if the quanta are not released independently), we represent the probability distribution of the event amplitudes as a gaussian distribution described by a mean amplitude \( m \) and a variance \( \sigma^2 \). We denote the mean amplitude of the different types of events as \( m_p, m_s, m_t \) and their variances by \( \sigma_{p}^2, \sigma_{s}^2, \sigma_{t}^2 \).

As is conventional for a gaussian distribution, any percentile can be depicted in terms of the standardized \( z \) measure. For example the 2.5\% percentile of a normal distribution lies at \( z_m = \frac{\sigma}{\sqrt{n}} = -1.96 \).

The PMs give information about the fraction of events of each type larger than a selected \( \theta \) that is, about the percentiles of the distribution. So, the number of missed P’s could be assessed in Fig. 3A1 by computing that of the overlong intervals (with \( I_{500} \gg \pi/P \)), which do not appear on the PMs due to the chosen scales. Since \( \pi/P \) was known, one could interpolate the position of the missing P’s. This procedure indicated 52 of them inferred among the 566, i.e., 92\%, detected ones.

Generally in our experiments, there was a threshold at which 95\% of P and no more than 5\% of T events were detected. This observation can be translated into the notation as: \( m_p + z_{0.05} \sigma_p > \theta \) and \( m_t + z_{0.05} \sigma_t > \theta \) (where \( z_{0.05} \) is negative). Combining the two statements we have

\[
m_p + z_{0.05} \sigma_p > m_t + z_{0.05} \sigma_t \tag{B1}
\]

where \( z_{0.05} \approx -1.6 \) and \( z_{0.95} \approx 1.6 \).

Similar relationships can be found by comparing other classes of events: the fact that approximately 50\% of S and 2\% of T are greater than \( \theta = 3.500 \) for the data of Fig. 3 indicates that \( m_s + z_{0.95} \sigma_s > m_t + z_{0.95} \sigma_t \) or, substituting in the tabulated values for \( z_{0.50} \) and \( z_{0.98} \),

\[
m_s > m_t + 2 \sigma_t.
\]

**Appendix B2: implications of \( np \) values on the percentage of detection**

\( m \) and \( \sigma^2 \) can be related to the release parameters \( n \) and \( p \). For each type of events and assuming binomial statistics, we have \( m = np \) and

\[
\sigma = \sqrt{np(1-p)} \tag{B1}
\]

Relationships of the form of Eq. B1 can be used to determine theoretical limits on the number of quanta potentially involved in each type of events. For any nonzero value of \( \sigma_p \) and \( \sigma_t \), this equation directly implies that \( m_p > m_t \) and therefore that \( n_p p > n_t P \). In other words, there can be no signal-flag pattern if the \( np \) products are equal. This rule can be generalized to a compound binomial. If the \( n \) quanta are released with different probabilities, then \( m_p = \sum_{s=1}^n p_k p_r \),

\[
m_T = \sum_{k=1}^n p_k P \cdot \sigma_T = \sqrt{n \sum_{k=1}^n p_k^2 (1-p_k^2)} \quad \text{and} \quad \sigma_T = \sqrt{n \sum_{k=1}^n p_k (1-p_k) P}.
\]

Then it is easily shown that for fixed \( n \) and \( m \) the compound binomial has always a smaller \( \sigma \) than the simple binomial distribution.

For both binomial-type models, the smaller are the values \( \sigma_p \) and \( \sigma_t \), the closer can be \( m_p \) and \( m_t \). In the theoretical limits where \( \sigma_p = 0 \) and \( \sigma_T = 0 \), it is required from Eq. B1 that \( n_p > n_t \).

Using the coefficient of variation \( CV \), we have from Eq. B1 the relation

\[
m_p > m_t \frac{1 + z_{0.05} CV_T}{1 + z_{0.05} CV_P} \tag{B2}
\]

where \( CV_p \) and \( CV_T \) are the coefficients of variation of the P and T events respectively. For example, taking \( CV_p = 0.28 \) which is the mode of pooled experimental data from M cell’s inhibitory connections (Korn et al. 1986), and assuming a similar \( CV \) for the S and T events, indicates that \( n_t p > 2.62 n_p p \). The observed relationships between P and S and T events indicate that \( n_t p > 1.81 n_p p \) and \( n_t P > 1.55 n_p P \).

The CV also constrains the relationship between \( n \) and \( p \)

\[
p \geq \frac{1}{CV^2 n} \tag{B3}
\]

where \( CV \) is the coefficient of variation and \( n \) is the number of events.
where the equality holds for a simple binomial model and the inequality for a compound binomial model (where $p$ is the mean of $p_k$). Even under these constraints, Eqs. B2 and B3 allow a wide range of physiologically reasonable $n$ and $p$ combinations consistent with the empirically observed PMs.

Appendix B3: extension to synchronized neurons

Considering $N$ cells firing synchronously, where cell $i = 1, \ldots, N$ has $n_i$ potential quanta for release, the total number $n$ of possible quanta is

$$n = \sum_{i=1}^{N} n_i$$

Denoting the probability of release of quantum $k$ in cell $i$ as $p_{ki}$, then the mean number of quanta released is

$$m = \sum_{i=1}^{N} \sum_{k=1}^{n_i} p_{ki}$$

which is exactly the same as if all $n$ quanta were from a single cell. The variance in the number of quanta released in the $N$ cells is

$$\sigma^2 = \sum_{i=1}^{N} \sum_{k=1}^{n_i} p_{ki}(1-p_{ki})$$

which again is identical to the situation where all $n$ quanta were from a single cell.

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