Kinematic and Kinetic Constraints on Arm, Trunk, and Leg Segments in Target-Reaching Movements

James S. Thomas,1 Daniel M. Corcos,2 and Ziaul Hasan3

1School of Physical Therapy, Ohio University, Athens, Ohio; 2Departments of Movement Sciences, Bioengineering, and Physical Therapy, University of Illinois at Chicago, and Department of Neurological Sciences, Rush Medical College, Chicago; and 3Departments of Movement Sciences and Physical Therapy, University of Illinois at Chicago, Chicago, Illinois

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Thomas, James S., Daniel M. Corcos, and Ziaul Hasan. Kinematic and kinetic constraints on arm, trunk, and leg segments in target-reaching movements. J Neurophysiol 93: 352–364, 2005. First published September 1, 2004; doi:10.1152/jn.00582.2004. We studied target reaching tasks involving not only the arms but also the trunk and legs, which necessitated some trunk flexion. Such tasks can be successfully completed using an infinite number of combinations of segment motions due to the inherent kinematic redundancy with the excessive degrees of freedom (DOFs). Sagittal plane motions of six segments (shank, thigh, pelvis, trunk, humerus, and forearm) and dynamic torques of six joints (ankle, knee, hip, lumbar, shoulder, and elbow) were analyzed separately by principal component (PC) analyses to determine if there was a commonality among the shapes of the respective waveforms. Additionally, PC analyses were used to probe for constraining relationships among the 1) relative magnitudes of segment excursions and 2) the peak-to-peak dynamic joint torques. In summary, at the kinematic level, the tasks are simplified by the use of a single common waveform for all segment excursions with 89.9% variance accounted for (VAF), but with less fixed relationships among the relative scaling of the magnitude of segment excursions (62.2% VAF). However, at the kinetic level, the time course of the dynamic joint torques are not well captured by a single waveform (72.7% VAF), but the tasks are simplified by relatively fixed relationships among the scaling of dynamic joint torque magnitudes across task conditions (94.7% VAF). Taken together, these results indicate that, while the effective DOFs in a multi-joint task are reduced differently at the kinematic and kinetic levels, they both contribute to simplifying the neural control of these tasks.

INTRODUCTION

Starting from an upright standing posture and reaching for a target that requires some forward bending of the trunk can involve many different configurations of the trunk and limb segments. This is due to kinematic redundancy, i.e., the fact that the number of joints involved in these reaching tasks furnishes more mechanical degrees of freedom (DOFs) than are required to complete this task. The resolution of kinematic redundancy, when it does not entail freezing some of the DOFs, calls for constraining relationships among them (Bernstein 1967). For example, it has been reported in earlier studies that the spatial trajectories of the wrist are largely unaffected by changes in speed and load (Soechting and Lacquaniti 1981) and that the velocity profile of the end effector tends to be bell-shaped (Flash and Hogan 1985). Additionally, coupling between the shoulder and elbow joints has been shown for circular drawing tasks (Soechting et al. 1986) and for reaching tasks (Soechting and Lacquaniti 1981). Invariant features of segment kinematics such as smooth trajectories and fixed coupling of joints in these two DOF reaching tasks suggest that the CNS could plan movement in kinematic parameter space.

Kinematic invariance can also be expressed through a common temporal kinematic waveform used by all the segments involved in a given movement task. The similarity of the shapes of time-series data can be brought out by the method of principal component (PC) analysis. This is a data reduction method that transforms a given set of mutually correlated variables into a new set of variables (components or factors) that are uncorrelated, and in which the first few variables account for most of the variance in the original data (Jolliffe 1986). Alexandrov et al. (1998), using PC analysis on the time-series kinematic data of the ankle, knee, and hip from individual subjects performing rapid trunk flexion tasks, found that, within any movement trial, >99% of the total angular variance could be accounted for by the first PC alone. Thus they showed that the kinematic waveforms of the three lower extremity joints were nearly identical in shape for the trunk flexion task. These findings were supported in a follow-up study by the same group (Alexandrov et al. 2001). Using a similar approach, others have also found a commonality in the shape of kinematic waveforms of the lower extremity joints in tasks involving gait (Bianchi et al. 1998; Borghese et al. 1996; Grasso et al. 1999; Mah et al. 1994; St-Onge and Feldman 2003). Use of a common kinematic waveform is one way that the CNS may reduce the complexity of multi-DOF tasks. However, it is not known whether a common kinematic waveform generalizes to a whole-body reaching task that necessitates motion of the lower extremities, the trunk, and the reaching limb.

Identification of invariant kinematic features in movement task has been interpreted as evidence that the CNS plans movement in a kinematic parameter space. Others, however, have proposed that the CNS may reduce the complexity of multi-joint tasks at the level of the dynamic joint torques (Gottlieb et al. 1996a,b). According to this idea, termed “linear synergy,” the CNS commands produce similarly shaped torque waveforms at the different joints, in addition to the torques needed for equilibrium, and the observed kinematic patterns simply emerge from the properties of the mechanical system (Gottlieb et al. 1996a,b). The experimental paradigms testing...
the linear synergy hypothesis have been limited to the shoulder and elbow torques in two and three DOF pointing tasks (Almeida et al. 2000; Gottlieb et al. 1996a; Zaal et al. 1999). Whether this same relationship holds in whole-body reaching tasks with larger numbers of DOFs is unknown.

To date, the results from human studies involving multi-joint reaching provide no clear evidence that the CNS explicitly plans movement in a kinematic or kinetic parameter space. Consistent with these findings, evidence from investigations of patterns of cortical activity in the primary motor cortex of nonhuman primates performing two and three DOF reaching tasks indicate that both kinematic and kinetic parameters are represented (Graham et al. 2003; Scott and Kalaska 1997; Sergio and Kalaska 2003). It may be the case that the complex role of the CNS in controlling multi-joint tasks may occur in another parameter space such as joint power (Scott et al. 2001).

While most investigations of kinematic redundancy have been restricted to two and three DOF reaching tasks, we have chosen to examine whole body reaching tasks that involve goal-oriented reaching to a target while maintaining postural equilibrium. Focusing on six DOFs of the leg, trunk, and arm segments in the sagittal plane, we showed in a previous paper that the rotational excursion of each segment depends not only on target location, but also on movement speed and on inter-subject differences (e.g., ankle dorsiflexion-knee flexion or ankle plantar flexion-knee extension movement strategy) (Thomas et al. 2003). The latter dependencies are made possible by kinematic redundancy. We now turn to the question of whether there are any relationships that are preserved among the motions of the different segments in the face of variations in target location, speed, and subject preference. We investigate this question by focusing not only on the magnitudes of the motions, but also on the shapes of their temporal waveforms. In addition, we explore the possibility that the constraining relationships may be better described in terms of the shapes and magnitudes of dynamic joint torques rather than segment motions.

Separate PC analyses were performed on the time series kinematic and kinetic waveforms as well as the magnitude of segment excursions from initial posture to target contact and the peak-to-peak dynamic joint torques. The criterion used for all the PC analyses was that the first PC must account for >90% of the total variance. Jolliffe (1986) suggests a cut-off criterion of 70–90% for PC analyses, and we have chosen the more conservative value of 90% for our analyses. Thus using the criterion above, the data were analyzed across all movement trials and all subjects to determine 1) the commonality of the temporal kinematic waveforms, 2) the commonality of the temporal kinetic waveforms, 3) the relationships among the relative magnitudes of segment excursions, and 4) the relationships among the relative magnitudes of the dynamic joint torques. The results of these analyses suggest that the CNS reduces the complexity of multi-joint movements both in kinematic and kinetic parameter spaces, although in different ways. The temporal kinematic waveforms are nearly identical in shape across all conditions, while the relative magnitudes of the segment excursions are more variable. In contrast, the temporal kinetic waveforms have significant variability in shape, but the relative magnitudes of the dynamic joint torques are highly linear.

**METHODS**

Twenty healthy normal individuals, 10 men (mean age, 25.5 yr; range, 21–38 yr) and 10 women (mean age, 25.9 yr; range, 20–37 yr), participated in this experiment. All participants signed informed consent, and the institutional review board of the University of Illinois at Chicago approved this study.

**Procedure**

The experimental procedures have been described in an earlier publication (Thomas et al. 2003). To summarize, barefoot subjects stood upright, with arms on the sides, and were instructed to “wait for the go signal and then reach for the target, touch the target with the right index finger, and maintain contact with the target until instructed to return to an upright posture.” The target was a small metal plate (3 × 4 cm) placed in front, and a light-emitting diode (LED) adjacent to the target was used for the “go” signal. Two target positions were employed in the para-sagittal plane, “low” and “high,” whose locations were based on the subject’s segment lengths, such that the target could have been reached in theory by flexing the shoulder by 90° and flexing the hip by 60 or 30°, respectively, with no changes in all other joint angles. Subjects were instructed to reach to the two target locations at self-selected “comfortable” and “fast” speeds, with some training at the latter speed to obtain a similar ratio of movement times (fast/comfortable = 0.42) for each of the two targets. Twelve fast speed trials were done in a block following 12 comfortable speed trials, and the target location was alternated within each block. Movements of six segments (shank, thigh, pelvis, trunk, humerus, and forearm) were recorded at 100 Hz using a two-camera Selspot system, with a pair of infrared light emitting diodes (IREDs) affixed to each segment.

**Kinematic and kinetic variables**

From the positions of the two IRED markers on a segment, we determined its sagittal plane orientation (φ) at each instant of time. The orientations of the six segments (shank, thigh, pelvis, trunk, forearm, and humerus) were calculated. These were defined as positive in a counterclockwise direction, as seen from the subject’s right side, with the anterior direction corresponding to 0°. Kinematic analyses were based on the six waveforms of the φs as functions of time.

For kinetic analyses, the joint torques—sometimes called generalized muscle torques—were derived from a sagittal plane inverse dynamics analysis. The gravitational components of the torques were excluded to yield the “dynamic” torques (Gottlieb et al. 1996a). A linked-segment model consisting of seven segments was used for the inverse dynamics calculations. The seven segments were defined as follows: the left and right feet, the left and right shanks, the left and right thighs, the pelvis (greater trochanter to L1), the trunk (comprised of the thorax from L1 to the 1st thoracic vertebrae, the head and neck, and the left humerus, forearm, and hand), the right humerus, and the right forearm and hand. The dynamic component of the joint torque was calculated for the following six joints: ankle, knee, hip, lumbar, shoulder, and elbow. Inverse dynamics calculation required estimates of the inertial parameters of the segments (mass, location of center of mass, and radius of gyration), which were approximated using the regression equations provided by Plagenhoef et al. (1983). We used the kinematic data and the location of the center of mass (COM) of each segment to determine the dynamic joint torque at each joint for each instant of time. First, for each segment, we calculated at every instant of time the orientation angle of the x-axis (horizontal), the positions of the two joints, and the position of the segment’s COM. These data were smoothed with a 61-point fourth-order Savitzky-Golay filter (Press et al. 1992). That is, at each sample time, fourth-order polynomials were fit in the least-squares sense, to
the data at that point and 30 neighboring samples on each side. The polynomial coefficients were then used to determine velocity and acceleration. Subsequently, we calculated at each instant of time, the dynamic joint torques of each joint in sequence from the elbow to the ankle using the angular accelerations, the accelerations of the COM, and Newtonian equations of motion (Winter 1990) (see Appendix for details). For the lower extremities, the mass of the left and right thigh were added to create a single thigh segment, and it was assumed that the movement paths of both thigh segments were symmetrical. This process was repeated for the shank segments to calculate the dynamic joint torques of the hip, knee, and ankle joints.

**PC analyses of shapes of time series waveforms**

Separate PC analyses were applied to the time series segment orientation data and the dynamic joint torque data. In each case, the attempt was to quantify the similarity in shapes of the waveforms across segments or joints and across all movement trials made to different targets, different speeds, and by different subjects. PCs can be based on the eigenvectors of either the covariance or the correlation matrix of the original data set. PC analysis based on the covariance matrix gives greater weight to the variables that vary over a larger range (Jolliffe 1986). Since we were interested in determining the similarity of the shape of the waveforms, PC analysis based on the correlation matrix was chosen. The use of a correlation matrix is equivalent to amplitude normalizing the waveforms, and it ensures that the analyses are not dominated by segments with the largest excursions or joints with the largest dynamic torques.

To determine the extent to which a common kinematic waveform was used for this whole body reaching task, we time-normalized the segment orientation data. First, for each movement trial, trunk velocity was calculated and movement onset was defined as the point where the velocity trace first rose 5% above baseline (relative to peak velocity) and offset the point where the velocity trace returned to within 5% of baseline. The trunk segment was used because its motions were large, smooth, and always in the same direction. The segment orientation data from onset to offset were time normalized to 150 points using an interpolation routine in MATLAB. The movement times ranged 490–2,500 ± 375.6 (SD) ms for the comfortable pace reaches and 200–780 ± 89.7 ms for the fast-paced reaches. Thus our data set consisted of 2,880 (20 subjects x 24 movement trials x 6 segments) waveforms, each 150 points in length. A correlation matrix of 2,880 x 2,880 was derived from this data set.

PC analysis was also performed on the correlation matrix derived from the time normalized dynamic joint torques of the ankle, knee, hip, spine, shoulder, and elbow joints of all subjects. For each trial, torque onset was determined as the point where ankle joint torque changed 5% of peak torque from baseline, and offset was defined as the point where ankle joint torque returned to within 5% of the baseline. Ankle joint torque was used for this purpose because this torque was large, smooth, and always in the same direction. The dynamic torque curves were time-normalized from onset to offset to equal 150 points using an interpolation routine in MATLAB. Thus our data set consisted of 2,880 (20 subjects x 24 movement trials x 6 joints) waveforms, each 150 points in length.

For each PC analysis of kinematic and kinetic waveforms, we derived a representative waveform, which we term an *eigencurve*, from the sum of the products of the scaling coefficients of the first PC (eigenvector) multiplied by the associated raw data (i.e., segment orientations or dynamic joint torques).

**PC analyses of magnitudes of segment motions and joint torques**

A PC analysis was used to determine the extent to which linear relationships exist among the magnitudes of segment motions across targets, speeds, and subjects. For this purpose, the PC analysis was based on the (6 x 6) covariance matrix of all segment excursions (6) across all movement trials (24) and all subjects (20). Of the total variance in these data, the largest contribution was from the change in orientation of the trunk (35%), and the smallest was from the change in orientation of the shank (6.4%). The six segment excursions were determined by taking the differences between the orientation angles (averaged over a 100-ms epoch) prior to the go signal and the orientation angles (averaged over a 100-ms epoch) following target contact. Likewise, the analysis of relationships among the magnitudes of the joint torques was based on finding eigenvectors of the covariance matrix in which, instead of the six segment excursions, we used the six peak-to-peak values of the dynamic joint torques. For these data, 99.64% of the total variance was contained in the four postural joints (i.e., ankle, knee, hip, and spine), with the remainder contained within the shoulder and elbow joints.

One can interpret the PCs obtained from the magnitude data geometrically as vectors in a six-dimensional (6D) space, each dimension corresponding to a segment excursion or joint torque magnitude, depending on whether the analysis was based on kinematic or kinetic variables. Each PC, then, is a vector in this space, which passes through the point of mean values of the magnitude data, and the six PCs are mutually orthogonal. If the first PC were to account for 100% of the variance in the data, one would conclude that the data are perfectly fit by a straight line in 6D space, and therefore, the six magnitudes are determined by a single variable. In other words, the six magnitudes are so constrained (by 5 relationships among them) that the specification of one of them specifies them all. If, on the other hand, two PCs are needed to account for most of the variance, it would follow that the data lie, not on a line, but on a 2D plane in the 6D space, and therefore, two variables are required to specify all six.

The orientation of the PC in 6D space provides further information about the coupling of the magnitudes. For example, if the PC vector passes through the origin of the 6D space, one could say that the six magnitudes are not only linearly related, but also that the ratios of their changes remain unaltered across the trials included in the analysis. To investigate this possibility, we determined the angle (in 6D space) between a PC and the vector joining the origin to the point whose six coordinates are the mean values (across 20 x 24 = 480 trials) of the magnitudes of the six segment excursions (or peak-to-peak dynamic torques). Given the PC vector and the vector through the origin, the angle between them was determined based on the definition of the scalar product. (The cosine of the angle equals the scalar product of the two vectors divided by the product of their magnitudes.) This angle would be zero if the PC went through the origin, implying proportionality among the magnitude changes.

**RESULTS**

Representative kinematic data of comfortable and fast paced movement trials of a subject reaching to the “low” target are presented in Fig. 1, A and B, and the corresponding stick figures are shown in Fig. 1, C and D, respectively. The initial segment orientation angles of the shank, thigh, pelvis, and trunk are close to +90°, i.e., nearly vertical, whereas the initial orientation angles of the humerus and forearm are nearly −90° and −75°, respectively; the negative signs indicate clockwise orientations with respect to the anterior direction. To reach the target, this subject rotated the shank, pelvis, and trunk clockwise from the initial posture, and the thigh, humerus, and forearm were rotated counterclockwise. Thus the changes in orientation angles (Δθs) of the shank, pelvis, and trunk were negative, whereas the changes in orientation angles of the thigh, humerus, and forearm were positive. The dynamic joint torques calculated from the movement trials shown in Fig. 1, A and B, are presented in Fig. 2, A and B. In general, the dynamic
torques are biphasic waveforms with initial negative peak deflections for the ankle, knee, hip, and spine and initial small, positive peak deflections for the shoulder and elbow joints. Between the comfortable and fast-paced movement trials shown in Fig. 2, A and B, the dynamic joint torque waveforms differ in magnitude as well as time course. The latter differences are brought out more clearly when the torque waveforms are normalized in magnitude, as is done in Fig. 2, C and D.

**Commonality of kinematic waveforms**

To determine the commonality of the shapes of the kinematic waveforms, PC analyses were performed on the correlation matrix of the time series data of the shank, thigh, pelvis, trunk, humerus, and forearm segments of an individual reaching for the low target in a comfortable pace movement trial. B: reaching for the low target in a fast-paced movement trial. Counterclockwise rotations as seen from the subject’s right side are shown as increases. C: stick figures plotted every 100 ms from movement initiation until target contact are derived from the same time series shown in A and the subject’s anthropometric data. D: stick figures derived from the same time series shown in B.

**Commonality of kinetic waveforms**

To determine the commonality of the shapes of the kinetic waveforms, PC analyses were performed on the comfortable-paced movement trials and the fast-paced movement trials. These analyses revealed that the first PC accounted for 89.5% of the total angular variance for the comfortable-paced movement trials and 90.2% for the fast-paced trials. The high value of the VAF indicates that across all segments, targets, and subjects, there exists a common waveform used to perform these reaching tasks, and that movement speed has only a small effect on the general shape of this common waveform. This point is shown graphically in Fig. 3, which depicts the eigencurves derived from these analyses. Visual comparison of the shapes of the time series kinematic waveforms presented in Fig. 1, A and B, to the eigencurves shown in Fig. 3 further shows the commonality of the kinematic waveforms.
FIG. 2.  A: time series dynamic joint torques of the ankle, knee, hip, spine, shoulder, and elbow of an individual reaching for the low target in a comfortable pace movement trial were derived from kinematic data presented in Fig. 1.  B: reaching for the low target in a fast-paced movement trial.  C: dynamic joint torque data from A with peak-to-peak amplitude normalized to 1.  Arrows highlight the peak torques of the reaching limb and postural joints and their zero crossings.  D: dynamic joint torque data from B with peak-to-peak amplitude normalized to 1.

FIG. 3. Kinematic waveforms closely approximate the waveforms of all segment orientations for all subjects and targets. The “eigencurves” depicted are derived for comfortable and fast-paced movement speeds. The variance accounted for by these eigencurves is 89.5 and 90.2%.
shoulder, and elbow dynamic joint torques across all movement trials for all subjects. This analysis revealed that the first PC accounted for only 72.7% of the total variance of the dynamic joint torques. This VAF falls considerably short of the 90% criterion level, which indicates that there is not a single common waveform whose different scalings correspond to the various torque waveforms. This conclusion supports the difference that was noted in connection with Fig. 2, comparing waveforms between the comfortable- and fast-paced conditions. We therefore considered the possibility that there may be one underlying waveform for the comfortable speed and another for the fast speed. Separate PC analyses of the comfortable- and fast-paced movement trials were performed, which, however, revealed that the first PC derived from each subset of data could account for only 69.3 and 78.8% of the respective variances. While the shapes of the torques waveforms are somewhat different for the two speeds, even when the data are stratified by movement speeds, the first PC does not meet the criterion threshold of 90% VAF for commonality of the shapes of the waveforms.

We considered the possibility that the unaccounted variance may be arising from timing differences among the torque waveforms at the different joints. Indeed, visual inspection of Fig. 2C (or Fig. 2D) reveals that the timings of the peak and of the zero crossing are quite different for the torques at the leg and trunk joints (i.e., ankle, knee, hip, and spine) compared with the joints of the reaching limb (i.e., shoulder and elbow). Formal analysis of the timing of the peak torques and zero crossings of the six joints reveals that there are significant differences between the two sets of joints. The times of peak torque and zero crossing of each joint were expressed as latencies relative to the ankle joint and analyzed using paired samples t-test. Peak torque latencies were significantly different for the shoulder and elbow joint torques ($P < 0.05$). On average, peak torque of shoulder and elbow preceded peak ankle torque by 117 and 125 ms, respectively (Fig. 4A). The

![Peak Torque Latencies](image)

**Fig. 4.** Time of the peak torque and 0 crossing of each joint expressed as a latency relative to the ankle joint. A: peak torque latencies were significantly different from the ankle only for the shoulder and elbow joint torques ($P < 0.05$). On average, peak torque of shoulder and elbow preceded peak ankle torque by 117 and 125 ms, respectively. B: for each joint torque except the knee, the zero crossing was significantly ($P < 0.05$) different from that of the ankle. On average, zero crossings of shoulder and elbow torques preceded zero crossing of the ankle by 136 and 154 ms, whereas the hip and spine zero crossings followed the ankle by 15 and 23 ms, respectively.
zero crossing latency was significantly \((P < 0.05)\) different from that of the ankle for each joint torque except the knee. On average, zero crossings of shoulder and elbow torques preceded zero crossing of the ankle by 136 and 154 ms, respectively, whereas the hip and spine zero crossings followed the ankle by 15 and 23 ms, respectively (Fig. 4B).

It is possible that the torque waveforms are similar in shape, but because some are delayed with respect to others, the PC analysis was unable to bring out their similarity. To address this possibility, we time shifted the dynamic joint torques so as to align their zero-crossing times, before normalizing them in time (as described in METHODS). PC analysis then revealed that the first PC accounted for 91.3% of the total variance of the dynamic joint torques. (In the subsets of comfortable and fast-paced movement trials, the VAF by the 1st PC was 88.4 and 94.6%, respectively. The corresponding eigencurves are shown in Fig. 5.) This analysis suggests that a common waveform does exist at the level of the dynamic joint torques, except that, unlike the case for the common kinematic waveform presented earlier, it is time shifted differently for the postural versus the reaching joints.

**Relationship among segment excursions**

PC analysis of the time series waveforms indicates the use of a common kinematic waveform across individual segments, targets, speeds, and subjects. However, there still exists the problem of apportionment of motion to the six segments in a kinematically redundant system. To try to identify a relationship among the magnitude of the motions apportioned to the six segments, PC analysis was performed on the covariance matrix derived from excursions of the six segments across all trials and all subjects. The results of this analysis yielded six PCs, which are listed below, each in terms of its six components. Each component, corresponding to a change in segment angle, is given as a coefficient multiplying a unit vector \(u\). The subscripts of \(u\) identify the segment: sh, shank; th, thigh; pl, pelvis; tr, trunk; hu, humerus; fa, forearm. (The + and − signs stand for vector addition/subtraction, and the coefficients are scaled so that the sum of their squares for each PC equals the eigenvalue.)

\[
\begin{align*}
PC1 &= (-1.52u_{\text{sh}} - 2.65u_{\text{th}} + 8.84u_{\text{pl}} + 19.39u_{\text{tr}} + 10.82u_{\text{hu}} + 10.94u_{\text{fa}}) \quad \text{[VAF = 62.2%]} \\
PC2 &= (-6.49u_{\text{sh}} + 13.23u_{\text{th}} - 0.61u_{\text{pl}} - 0.45u_{\text{tr}} + 0.96u_{\text{hu}} + 2.63u_{\text{fa}}) \quad \text{[VAF = 20.0%]} \\
PC3 &= (3.73u_{\text{sh}} + 0.49u_{\text{th}} - 8.76u_{\text{pl}} + 0.35u_{\text{tr}} + 3.71u_{\text{hu}} + 3.44u_{\text{fa}}) \quad \text{[VAF = 10.4%]} \\
PC4 &= (0.68u_{\text{sh}} + 1.17u_{\text{th}} - 1.93u_{\text{pl}} + 4.47u_{\text{tr}} - 3.37u_{\text{hu}} - 2.66u_{\text{fa}}) \quad \text{[VAF = 3.9%]} \\
PC5 &= (2.22u_{\text{sh}} + 0.71u_{\text{th}} - 0.99u_{\text{pl}} - 0.37u_{\text{tr}} - 2.84u_{\text{hu}} + 3.14u_{\text{fa}}) \quad \text{[VAF = 2.2%]} \\
PC6 &= (2.58u_{\text{sh}} + 1.49u_{\text{th}} + 1.19u_{\text{pl}} - 0.09u_{\text{tr}} + 1.31u_{\text{hu}} - 1.37u_{\text{fa}}) \quad \text{[VAF = 1.3%]} 
\end{align*}
\]

It is clear, then, that three variables are sufficient to specify all six segment excursions to 92.6% accuracy. The effect of each of the three variables on the six excursions can be shown with the aid of stick figures. A series of stick figures of final configurations were generated, assuming that only one of the PCs was operative, by assigning different arbitrary weights (for example, \(-2, -1, 0, 1, 2\)) to a given PC, while giving zero weight to all other PCs. The initial configuration was assumed to be upright with the arms at the side. The final configurations depicted in Fig. 6A are based on various weights assigned to the loading coefficients of PC1 alone, Fig. 6B is based on PC2 alone, and Fig. 6C is based on PC3 alone. The fact that these three PCs account for 92.6% of the variance in the data implies that every observed final configuration can be reasonably well approximated as a combination of three stick figures, taken from each of the Fig. 6, A–C.

Visual inspection of Fig. 6A reveals that the final configurations of the trunk and arm segments are captured by PC1. (This reflects the fact that the components of PC1, given above,
have the largest coefficients for the trunk, pelvis, humerus, and forearm. In contrast, Fig. 6B shows that the loading coefficients of PC2 primarily capture the final configurations of the thigh and shank. (This is because the thigh and shank have the largest coefficients in the expression for PC2.) This captures either a knee flexion/ankle dorsiflexion strategy or a knee hyperextension/ankle plantar flexion strategy. Figure 6C shows that PC3 is not dominated by any specific segment excursion.

As stated previously, each PC is a vector in 6D space. The variance it accounts for is a reflection of how closely the data are oriented along the vector, and the coefficients define the orientation of the vector in space. As explained in METHODS, we determined whether PC1 is so oriented as to pass through the origin of the 6D space, which would imply that in the configurations generated by this PC the ratios of changes in segment excursions would remain unaltered. The orientation of PC1, whose components were listed above, is at an angle of 76.2° with respect to the line joining the mean point of the segment excursions and the origin of the 6D space (i.e., 0 excursions). Thus PC1 does not go through the origin of this space. Although it is impossible to show a graph in six dimensions, this point can be shown in a 3D plot (Fig. 7A). The segment excursions of the pelvis, trunk, and humerus from all 480 movement trials are plotted. While the choice of which three segments to plot is arbitrary, we chose to plot the segment excursions of the pelvis, trunk, and humerus to display the relationship of the reaching limb and the postural segments. The vector formed by the first PC, which necessarily goes through the mean point of the segment excursions of the pelvis, trunk, and humerus for the 480 movement trials, is shown. Shown as well is the vector from the origin of this space to the mean segment excursions of the pelvis, trunk, and humerus, which is far from collinear with PC1.

**Relationship among peak-to-peak dynamic joint torques**

We now seek to determine the strength of the relationship among the magnitudes of the peak-to-peak dynamic joint torques from the six joints. This relationship was assessed by performing a PC analysis on the peak-to-peak excursions of the six dynamic joint torques across all trials and all subjects (480). As was the case with the analysis of the time series waveform, we used a covariance matrix for the PC analysis. The results of this analysis yielded six PCs, which are listed below, each in terms of its six components. Each component, corresponding to peak-to-peak joint torque, is given as a coefficient multiplying a unit vector. The subscript of u identify the joint: ank, ankle; kne, knee; hip, hip; spi, spine; shd, shoulder; elb, elbow:

PC1 = (30.49 u_ank - 27.43 u_hip - 6.72 u_elb - 2.06 u_shd + 0.47 u_hip)  [VAF = 94.7%]  
PC2 = (4.79 u_ank + 3.59 u_hip - 7.44 u_elb + 0.69 u_shd + 0.91 u_hip - 0.01 u_shd)  [VAF = 4.2%]  
PC3 = (1.74 u_ank - 2.51 u_hip + 0.31 u_elb + 0.59 u_shd - 2.27 u_hip - 0.01 u_hip)  [VAF = 0.7%]  
PC4 = (0.68 u_ank - 1.16 u_hip - 0.05 u_elb + 0.36 u_shd + 1.71 u_hip - 0.02 u_hip)  [VAF = 0.2%]  
PC5 = (-0.05 u_ank - 0.05 u_hip + 0.18 u_elb - 1.07 u_shd + 0.24 u_hip - 0.35 u_hip)  [VAF = 0.06%]  
PC6 = (-0.01 u_ank - 0.00 u_hip + 0.01 u_elb - 0.04 u_shd - 0.01 u_hip + 0.15 u_hip)  [VAF = 0.04%]  

The analysis revealed that the first PC accounts for 94.7% of the total variance of the peak-to-peak dynamic joint torques. Thus given the target, speed, and subject, only one variable needs to be specified (i.e., any 1 of the scalings of PC1), which is sufficient to reasonably determine all six torque magnitudes, all of which are linearly related to one variable.

The fact that a strong linear relationship exists among the six dynamic joint torques for these movement tasks leads to the question of whether they are all nearly proportional to each other. To answer this question, the angle was determined between the PC1 vector and the line joining the origin to the...
mean point. This angle was found to be 4.1°, which indicates that PC1 nearly passes through the origin of this space, and thus the relationship among the dynamic joint torques is not only linear, but proportionate as well. Once again, although it is impossible to show a graph in six dimensions, this point can be shown in a 3D plot (Fig. 7). The peak-to-peak dynamic joint torques of the hip, spine, and shoulder joints from all 480 movement trials are plotted. Visual inspection of this figure clearly indicates that the orientation of the first PC is nearly through the origin of this space. This indicates that there is a proportional relationship among the peak-to-peak dynamic joint torques across movement conditions.

**DISCUSSION**

Given the large number of DOFs involved in a free-standing reaching task, there are an infinite number of movement patterns that can be used to successfully reach a target due to the inherent kinematic redundancy. Nonetheless, we have identified constraining relationships at the level of both kinematic and kinetic variables in these multi-DOF tasks. In summary, at the kinematic level, a common waveform was used for all segment excursions, with less fixed relationships among the relative scaling of segment excursions. At the kinetic level, the dynamic joint torques are not well captured by a single waveform, but there were relatively fixed relationships among the scaling of dynamic joint torque magnitudes across task conditions. Taken together, these results indicate that the effective DOFs in a multi-joint task are reduced differently at the kinematic and kinetic levels. These constraining relationships may be interpreted as one way in which neural control of multi-joint tasks is simplified.

**Kinematic waveforms**

We found that we could identify a single kinematic waveform that could account for 89.6% of the total angular variance of the segment motions across target locations, movement speeds, and individual preferences. Furthermore, stratifying the data by movement speed had negligible effect on the VAF or on the commonality of the shape of the waveform (Fig. 3). While the VAF by the first PC is less than that reported by Alexandrov et al. (1998), who found that the VAF by the first PC was, on average, 99.7%, these differences are most likely due to task and methodological differences in our respective studies. First, our task involved six DOFs in contrast to the three DOF tasks used by Alexandrov et al. (1998). Second, we performed a PC analysis on the correlation matrix derived from the time normalized segment motion data from all movement trials, whereas Alexandrov et al. (1998) performed a separate PC analysis for each movement trial, and the PC analyses used the covariance matrix of the time series joint angle data. PC analysis of time series waveforms using the covariance matrix is dominated by the segment with the largest excursion (in this case the trunk) and results in greater VAF. Thus while our analytic approach is more conservative, we still find that a single waveform can account for nearly 90% of the total angular variance. We interpret our findings as a clear extension of the work of Alexandrov and colleagues, and conclude that the use of a common kinematic waveform does generalize to whole body reaching tasks that necessitate motions of the trunk, lower extremity, and reaching arms.

Taking the first derivative of the eigencurves shown in Fig. 3 would result in a smooth, bell-shaped velocity profile. Given that the eigencurve is representative of the time course for all six segments across all movement conditions and subjects implies that the bell-shaped velocity curve is an invariant feature for these six DOF tasks. Bell-shaped velocity profiles
have been reported for two-joint planar reaching tasks (Flash 1987; Hogan et al. 1987; Morasso 1981), unrestrained vertical arm movements (Atkeson and Hollerbach 1985), and sitting reaching tasks that necessitate some displacement of the trunk (Archambault et al. 1999; Ma and Feldman 1995; Wang and Stelmach 1998). Hollerbach and Flash (1982) have proposed that an invariant velocity profile reduces the complexity of computing the dynamic joint torques to a simple matter of uniformly scaling in time and amplitude the velocity profile. While we have found that simple speed scaling is not strictly adhered to in these standing reaching tasks (Thomas et al. 2003), the use of a common kinematic waveform across task conditions and subjects may yet reduce the complexity of these tasks.

Apportionment of segment excursions

Even with a common kinematic waveform that defines the time courses of segment motions, a nontrivial problem for the CNS is how to apportion the motions among the segments used to perform these reaching tasks. We found that PC1 and PC2 could account for >80% of the total angular variance of the magnitude of segment excursions across all experimental conditions and all subjects and that the addition of PC3 increased the VAF to >90%. These findings are consistent with our previous investigation of whole-body reaching tasks (Hasan and Thomas 1999) and suggest that the majority of the points representing segment excursions cluster near a plane in 6D space (Fig. 7A). If the points were to lie precisely in the plane, one could rigorously conclude that only two quantities (corresponding to the coefficients of PC1 and PC2) are required to specify the six segment excursions for each movement trial. While our criterion of 90% VAF by the first PC is not met for the relative scaling of segment excursions in these tasks, the data indicate that this six DOF problem is reduced to three DOFs through constraining inter-relationships among the magnitudes of the six segment excursions used to perform these reaching tasks.

If, for example, we examine the modules depicted in Fig. 6, A–C, which corresponds to various weights applied to the loading coefficients to PC1–PC3, we can see that PC1 is primarily responsible for the movement strategy used to determine excursions of the trunk and reaching limb. PC2, on the other hand, captures the relationships of the shank and thigh for these reaching tasks. That is, an individual uses an ankle dorsiflexion/ knee flexion strategy or an ankle plantar flexion/ knee hyperextension strategy. Consistent with this notion, we have previously reported that, in these whole body reaching tasks, the movement strategy of the shank and thigh are affected by task variables such as movement speed as well as individual preferences (Hasan and Thomas 1999; Thomas et al. 1998, 2003). Thus applying the appropriate weights and taking the linear combination of PC1 and PC2 will result in excursions of the trunk, reaching limb, and lower extremities that will bring the hand in contact with the target. PC3 is not dominated by any specific segment and may be related to small adjustments of segment excursions associated with this multi-DOF task.

PC analyses of the segment excursions reveal intersegmental correlations that may reduce the complexity of neural control of these multi-joint reaching tasks. However, the ratios of the changes in segment motions are not independent of target location or speed or intersubject differences. Thus the reduction in the complexity of the task at the level of kinematic excursions is not achieved by a simplistic scaling of all the segment motions.

Kinetic waveforms

Visual inspection of Fig. 2, C and D, reveals that the time series kinetic waveforms were 1) biphasic, 2) their shape was influenced by movement speed, and 3) regardless of movement speed, there were clear differences in timing comparing the leg and trunk joints with the joints of the reaching limb. Thus no single waveform could account for >90% of the total variance of the dynamic joint torque waveforms. This was primarily due to large differences in the timing of the joint torques of the reaching arm and the other joints. The differences in timing of the peaks and zero crossings of the dynamic joint torques was an unexpected finding based on literature of anticipatory postural adjustments (see review by Massion 1992). In the framework of anticipatory control, one would expect the dynamic torques of the “postural” joints to precede those of the reaching limb, which is the opposite of what we found for these tasks. However, we were unable to find similar examples in the anticipatory control literature that compared the timing of peak torques and zero crossings at different joints.

The large differences in timing of the joint torques of the reaching arm (i.e., shoulder and elbow) and the other joints (i.e., ankle, knee, hip, and spine) are not predictable from the linear synergy hypothesis put forth by Gottlieb et al. (1996a). However, the relatively small differences in timing of peak torque and zero crossing within the reaching arm (7, 13 ms) and within the other joints (13, 17 ms) indicate support for the linear synergy hypothesis as applied separately to the two sets of joints. Moreover, once we aligned the torque data of all joints by the zero crossings, the VAF was >90%. Overall, the data suggest separate timing of relatively synchronized torque pulses for the reaching joints and the other joints in our full-body reaching task. While this suggests a simplification of control of these multi-joint tasks at the kinetic level, one must consider how much these findings are driven by the mechanics of this task.

Given that the mass of the trunk (including the head, neck, and left upper extremity) is ~70% of the total body mass, the mass of the lower extremities would only account for <30% of the total body weight. The shapes of the dynamic joint torques of the ankle, knee, and hip would therefore be very similar because the mass of the system is largely concentrated in the trunk. The magnitudes of the dynamic joint torques for these three joints will, of course, be largest for the ankle, and progressively less so for the knee and hip, simply due to the increasing perpendicular distances of each of these joints from the acceleration vector of the trunk. Furthermore, the shapes of the dynamic torques would not be exactly alike since each segment has its own mass, which in part, contributes to the shape of the torque profile at that joint. However, there is no obvious mechanical reason for the dynamic joint torques at the shoulder and elbow to have a similar shape as the lumbar torque profile. While the motion of the trunk will have an effect
on dynamic joint torques needed at the shoulder and elbow, it is not the same dominant effect seen at the hip, knee, and ankle. The approximate similarity of the dynamic torque profiles across all joints may represent a coordination rule used to simplify the control of a multi-joint movement.

Apportionment of dynamic joint torque magnitudes

We have identified a strong linear relationship among the amplitudes of the six dynamic joint torques in these whole-body reaching tasks, as indicated by the fact that the VAF by PC1 was >93%. However, it should be noted that this relationship is dominated by the four postural joints because of the extremely small variance of the shoulder and elbow torques seen in these reaching tasks. Additionally, we have found that the magnitude ratios of the six dynamic joint torques were invariant not only to changes in movement speed, but also to changes in target location. The orientation of the vector that defines the linear relationship in 6D space goes nearly through the origin (Fig. 7B), which implies an invariance of the ratios to all the task manipulations in this experimental paradigm. In contrast, the “linear synergy” hypothesis posits that, while the ratio of dynamic joint torques remains invariant to changes in movement speed, it does change in response to changes in target location (Gottlieb et al. 1996a,b, 1997).

The differences between our findings and those of Gottlieb et al. (1997) could be due to the small number of targets in our study compared with theirs. Another possible reason for the discrepancy could be that in the series of experiments by Gottlieb et al. (1996a, 1997), subjects were limited to two DOF movements, which meant that for each target location there was a unique set of joint angles for the shoulder and elbow. It therefore follows that the ratio of joint torques in their experiments would have had to change with target location. In our six DOF reaching tasks, however, the specific orientations of each segment are not uniquely defined by the target location. Therefore the CNS can use a set of coordination rules of control in which the ratios of the torque magnitudes among the six joints are not obliged to change across target locations. Indeed, there are precedents in the literature for the near invariance of joint torque ratios in tasks with redundant DOFs. Consistent knee/ankle and hip/knee moment ratios have been reported for the recovery of stance posture in response to perturbations of various magnitudes (Eng et al. 1992; Yang et al. 1990). Eng et al. (1992) found invariant hip/knee moment ratios of 2:1 for postural control during rapid arm flexion tasks. Our findings could also be interpreted as an expansion of these findings.

Cortical studies

Evidence from investigations of nonhuman primates performing two and three DOF reaching tasks indicate that both kinematic and kinetic parameters spaces are coded in the primary motor cortex (Graham et al. 2003; Sergio and Kalaska 1998). While many cells in the primary motor cortex have been shown to be strongly modulated by all major parameters in these reaching tasks (Scott and Kalaska 1997), Cabel et al. (2001) found that single neurons were involved with controlling patterns of joint torques at multiple joints (i.e., shoulder and elbow joints). An interesting question in light of our present findings is whether one could identify single neurons associated with patterns of joint torques in a six DOF reaching task. In this study, the relative invariance of the ratios of joint torques across task conditions could be the result of these types of neurons that appear to be responsible for controlling multi-joint motor patterns. A significant challenge is to measure cortical activity in nonhuman primates performing reaching tasks that necessitate control of greater DOF. With respect to our finding that kinematic time series were more invariant than the dynamic torque time series data, there are no parallels in the cortical literature that could explain this phenomenon. However, it has been shown that the temporal profile of cortical activity is quite different between isometric force production and reaching movements (Graham et al. 2003; Sergio and Kalaska 1998). In contrast to the idea that kinematics and/or kinetics are explicitly represented in the nervous system, Todorov and Jordan (2002) have proposed that reductions in kinematic and kinetic parameters in a variety of movement tasks indicate that the CNS operates like an optimal feedback controller in performing coordinated multi-joint tasks (Todorov and Jordan 2002). They provide evidence through simulations and empirical observations that many simplifying rules and findings of invariant features in motor tasks can be predicted using task-optimal controller laws. They conclude that the observed invariant features are simply emergent properties of the system instead of computational shortcuts for the motor system.

In conclusion, according to Bernstein (1967), the CNS may resolve the problem of kinematic redundancy by reducing the effective DOFs in a coordinated task. We have provided evidence that the coupling of the DOFs occurs at both the kinematic and kinetic levels. While we are unable to resolve the question of whether movement planning occurs in kinematic or kinetic terms, we find that a similar biphasic, dynamic torque pulse is produced at all the joints, albeit with differences in latencies for the postural versus the reaching joints. As the task constraints are varied, the peak-to-peak amplitudes of the biphasic torque pulses at the different joints vary together. The kinematics that emerge display smooth changes in segment orientation, with all segments appearing to follow a similar velocity profile. The pronounced differences in the relative scaling of the segmental motions when task constraints are altered are therefore associated with subtle changes in the shapes and relative amplitudes of the dynamic torque profiles.

Appendix

Inverse dynamics equations

The horizontal axis is defined as the x-axis and the vertical axis as the z-axis. First, we calculate for each segment at every instant of time, the orientation angle ($\phi$) with respect to the x-axis, the positions of the two joints, and the positions of the COM of the segment. The position data are filtered with a fourth-order 61-point Savitzky-Golay filter. We double differentiate the filtered orientation data as well as the COM coordinates (in the x and z directions) for each segment at every instant in time.

For the inverse dynamic calculations, we proceed, at each instant of time, sequentially from the forearm/hand segment to the shank segment. For a specific segment, let $N$ be the joint near the hand, and $F$ the joint far from the hand. (e.g., for the thigh, $N$ is the pelvis and $F$ is the knee). We define the following distances in Fig. A1.
hand segment, torques are defined positive counterclockwise. (Note: for the forearm/R
Principle, we first introduce inertial forces (and torque) that are
to balance what we have just calculated. These are nothing but
Now, according to d’Alembert, all we need is to apply forces and
torques to point F on the segment FN.

Because this segment is moving, and we want to use d’Alembert’s Principle, we first introduce inertial forces (and torque) that are negative of mass times acceleration (and moment of inertia times angular acceleration). These are shown below in Fig. A3; assuming \( m \) is the segment mass and \( I \) is its moment of inertia about its COM.

Now we transfer to the point \( F \) both sets of forces and torques depicted above. The forces just get shifted, and each such shift adds a torque depending on the perpendicular distance by which the force is shifted. Thus the following forces and torques obtained by this shifting are equivalent to the forces and torques acting at point \( N \) as well as the inertial forces and torques.

Where

\[
\begin{align*}
R_{nx} &= R_n - m\vec{C}_x \\
R_{nz} &= R_n - m\vec{C}_z \\
T_r &= T_r - (R_{n_x} D_x) + (R_{n_z} D_z) + (m\vec{C}_x B_y) - (m\vec{C}_z B_x) - (I\dot{\phi})
\end{align*}
\]

Now, according to d’Alembert, all we need is to apply forces and torques to balance what we have just calculated. These are nothing but

\[ -R_{nx} - R_{n_z}, \text{ and } -T_r \text{ needed at the far joint } F \text{ to produce the observed motion (see Fig A4). We are now done with this segment. For the next sequential segment, we start with its near end and apply the negative of the forces and moment applied to the adjacent joint (Newton’s 3rd law), which of course means that the new segment’s near end will have the forces \( R_n \) and \( R_z \) and the moment \( T_r \) acting on it, as already calculated. We calculate what we need at its far end, using the same algorithm as before. And so on for all the segments. \]

**References**


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