Signal Transmission Between Gap-Junctionally Coupled Passive Cables Is Most Effective at an Optimal Diameter

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Nadim, Farzan and Jorge Golowasch. Signal transmission between gap-junctionally coupled passive cables is most effective at an optimal diameter. J Neurophysiol 95: 3831–3843, 2006; doi:10.1152/jn.00033.2006. We analyze simple morphological configurations that represent gap-junctional coupling between neuronal processes or between muscle fibers. Specifically, we use cable theory and simulations to examine the consequences of current flow from one cable to other gap-junctionally coupled passive cables. When the proximal end of the first cable is voltage clamped, the amplitude of the electrical signal in distal portions of the second cable depends on the cable diameter. However, this amplitude does not simply increase as the cable diameter is increased, as expected from the larger length constant; instead, an optimal diameter exists. The optimal diameter arises because the dependency of voltage attenuation along the second cable on cable diameter follows two opposing rules. As cable diameter increases, the attenuation decreases because of a larger length constant yet increases because of a reduction in current density arising from the limiting effect of the gap junction on current flow into the second cable. The optimal diameter depends on the gap junction resistance and cable parameters. In branched cables, dependency on diameter is local and thus may serve to functionally compartmentalize branches that are coupled to other cells. Such compartmentalization may be important when periodic signals or action potentials cause the current flow across gap junctions.

INTRODUCTION

A number of recent publications have reported the presence of gap-junctional coupling between many neuronal types that had not previously been observed (Connors and Long 2004). These studies highlight the growing importance attributed to gap junctions in the computations performed by many regions of the nervous system. The role of gap junctions in the generation and failure of neuronal oscillations and synchrony has been the subject of many theoretical studies (Bem and Rinzel 2004; Kepler et al. 1990; Kopell and Ermentrout 2004; LeBeau et al. 2003; Sherman and Rinzel 1991; Traub et al. 2003). Additionally, several studies have addressed the interaction between gap-junctional strength and membrane properties in action potential propagation, failure, and synchronization (Joyner et al. 1984; Keener 1990; Pfeuty et al. 2003). Processes of different dimensions are known to be electrically coupled to each other and the effectiveness of signal transmission between them may vary as a consequence of electrical load and coupling strength. For example, using modified cable theory, it has been shown that the diameter of gap-junctionally coupled cablelike processes, such as active cardiac myocytes, asymmetrically affects propagation velocity (Keener 1990). The role of process diameter on signal transmission in a single sealed-end cable has been examined with current and conductance inputs (Holmes 1989). However, the effect of diameter on signal transmission between processes coupled by gap junctions has not been studied.

We previously showed that gap-junctional coupling can affect the measurements of ionic conductances and that these effects are sensitive to the location and strength of the gap junction (Rabbah et al. 2005). The results of that study implied that electrical signaling between neurons becomes more effective if the gap-junctional coupling strengthens or if the neurons become more electrotonically compact. Here we report that this intuition is not entirely correct. We observe a nonmonotonic dependency of signal transfer between coupled passive cables as a function of cable diameter that is different in several respects from the single cable condition described by Holmes (1989). Although electrotonic access to a coupled cell improves with increased diameter, signal transfer may not. Signal transfer is maximized at an optimal cable diameter that depends differently on the cable properties and membrane properties of each cable and also sensitively depends on the coupling conductance. Thus signal transfer actually deteriorates as the coupled cables become more electrotonically compact past a certain optimal diameter. In branched cables the optimal diameter depends on the specific properties of individual branches of the postjunctional cell. Therefore functional compartmentalization may arise simply as a result of a difference in cable diameters in branches of coupled neuronal processes. We derive analytical expressions for signal transfer at steady state and examine the transient cases numerically. A comparison of the predicted optimal diameters with actual measurements of diameters of electrically coupled dendrites (Fukuda and Kosaka 2003) suggests that cable diameters may indeed be regulated to maximize neuronal signal transmission. Our results reveal a hitherto unknown phenomenon, that is, the existence of an optimal diameter for gap-junctional signaling between cablelike structures such as neuronal processes and muscle tissues.

METHODS

Analytical solutions

The steady-state equations for two cables coupled by nonrectifying gap junctions are derived from basic cable theory (Rall et al. 1995).
The steady-state voltage (calculated as deviation from the resting potential) at any distance \( x \) along a uniform cylindrical cable of finite length \( l \) and length constant \( \lambda \) can be calculated as

\[
V_x = V_0 \frac{\cosh (L - X) + \frac{R_e}{R_i} \sinh (L - X)}{\cosh L + \frac{R_e}{R_i} \sinh L}
\]

where \( L = l/\lambda, X = x/\lambda, R_e \) is the input resistance of a semiinfinite cable, \( R_i \) is the terminating resistance at \( X = L \) and \( V_0 \) is the voltage imposed at position 0 of the cable under voltage-clamp conditions. To calculate the current flow between the finite cables electrically coupled at the end (\( X = L \)) we determine the voltage at the end of cable 1. For a single uncoupled cable, at \( X = L \), Eq. 1 simplifies to

\[
V_L = \frac{V_0}{\cosh L + \frac{R_e}{R_i} \sinh L}
\]

with

\[
\lambda = \sqrt{\frac{R_e}{4R_i}} \quad \text{and} \quad R_e = 2 \frac{R_i R_c}{\pi d^{2.5}}
\]

\( R_e, R_i, \) and \( d \) have their usual meaning of specific membrane resistance (\( \Omega \text{cm}^2 \)), specific internal resistivity (\( \Omega \text{cm} \)), and diameter, respectively.

**End-to-end coupled cables**

The terminating resistance of cable 1 (\( R_{1T} \)), when coupled end to end to cable 2, is simply the sum of the gap-junction resistance \( R_c \) and the input resistance of cable 2 (\( R_{2i} \)) at the site of the gap junction. In general, for \( n \) end-to-end coupled cables, the terminating resistance of each cable is given by

\[
R_{1T} = R_{n_{m+1}} + R_c + R_{2i}
\]

with \( k = 1, \ldots, n - 1 \). For the last (sealed) cable \( R_{TN} = \infty \). The input resistances \( R_{n_{m},k} \) of all cables \( k = 1, \ldots, n - 1 \) are given by the standard form of the input resistance of a cable with an arbitrary terminating resistance \( R_{1T} \).

\[
R_{n_{m},k} = R_{n_{m}} + R_c \tanh \frac{L}{R_{n_{m}}} + R_{2i} \tanh \frac{L}{R_{2i}}
\]

where \( R_{n_{m}} \) and \( L \) are the input resistance of the semiinfinite cable and electrotonic length of cable \( k \), respectively, and \( R_{m,\infty} = R_c \coth L \).

By combining Eqs. 1 and 3 we obtain an expression for the voltage at any position \( X \) along cable \( k \)

\[
V_{k,X} = V_{k,0} \frac{\cosh (L_k - X) + \frac{R_e}{R_i} \sinh (L_k - X)}{\cosh L_k + \frac{R_e}{R_i} \sinh L_k}
\]

For \( k = 1 \), \( V_{1,0} = V_0 \) is the clamped voltage.

The beginning of cable \( k + 1 \) proximal to the gap junction behaves as a node in which the current flowing from the end of cable \( k \) through the gap junction is equal and opposite to the current flowing into cable \( k + 1 \) (i.e., \( I_{k+1} = I_{k+1,0} = 0 \)). This equation can be expanded to \( (V_k + 1,0 - V_{k+1,0})/R_i + V_{k+1,0} = 0 \). Solving for \( V_{k+1,0} \), using \( V_{k,X} = V_{k,T} \) and \( X = L_k \) we obtain

\[
V_{k+1,0} = \frac{R_{n_{m+1},k}V_{k,T}}{R_i + R_{n_{m+1}}}.
\]

Finally, the voltage at any electrotonic distance \( X \) along the sealed end cable \( n \) is given by Eq. 5 with \( R_e = \infty \)

\[
V_{n,X} = V_{n,-} \frac{\cosh (L_n - X)}{\cosh L_n}
\]

**Coupling along the middle region of semiinfinite cable pairs**

To study the dependency of signal transmission between the equivalent of axons coupled by gap junctions at any position along their length, as suggested by recent studies (Schnitz et al. 2001; Traub et al. 1999, 2003), we considered identical semiinfinite cables with a gap junction at a distance \( X = x/\lambda \) from the clamped end of cable 1. Cable 1 is clamped at the beginning of the cable. Thus its input resistance \( R_{m_{1}} \) is given by Eq. 4 using the terminating resistance \( R_{1T} \) given by

\[
R_{1T} = \frac{(R_i + R_{n_{2i}})R_1}{R_i + R_{n_{2i}} + R_1}
\]

which corresponds to two parallel resistors, one with value \( R_{n_{1}} \) and the other with value \( R_{n_{2}} \). \( R_{n_{2i}} \) is the input resistance of cable 2 at the gap-junction coupling position \( x \) and can be calculated as the equivalent resistance of two parallel resistors with values \( R_{n_{1}} \) and \( X \). The voltage of cable 1 at the coupling position \( X \) \( (V_{1,T}) \) is given by Eq. 2 with \( X = X \) substituting for \( L \) (with \( R_{m_{2i}} \) substituting for \( R_{2i} \) in Eq. 3). The voltage of cable 2 at the coupling position \( X \) is given by Eq. 6 with \( V_{1,T} \) substituting for \( V_{1,T} \) and \( R_{m_{2i}} \) for \( R_{m_{2}} \). The voltage \( V_{2,T} \) at the sealed end of cable 2 is given by Eq. 7a with \( L_3 = X \); \( V_{2,0} = V_{2,0} \).

**Two sealed cables coupled in the middle**

The treatment for sealed-ended identical cables coupled at middle positions \( X \) is similar to the semiinfinite case described above but the terminating resistance at position \( X \) is given by

\[
R_{m_{1},x} = R_{n_{1}} \coth (L_1 - X)(R_i + R_{n_{2i}})
\]

with input resistance \( R_{m_{2i}} \) given by

\[
R_{m_{2i}} = R_{n_{2i}} \coth (L_2 - X)\coth (L_1 - X)
\]

**Simulations**

Three numerical models were used in this study.

**MODEL 1.** A spiking isopotential neuron was built using standard Hodgkin and Huxley (1952) (H-H) equations coupled with a nonrectifying gap junction to the center of a passive cable 3,100 \( \mu \)m long divided into 100-\( \mu \)m-long segments with \( R_{m}, R_i \), and \( C_m \) as above. Integrations of membrane and cable equations were performed using...
MODEL 2. Two cells were coupled by a gap junction at their tips of their dendrites. Cell 1 was made of a spiking axon (six compartments of length 100 μm), 10 μm in diameter, built using standard H-H equations (Hodgkin and Huxley 1952) connected to a passive soma with surface 400π μm². Six passive dendrites of different diameters and length 600 μm (made of six compartments, each of length 100 μm) emerge from the soma, and an action potential was elicited with a 10-ms, 1-nA pulse into the tip of the axon. The end of the dendrite of diameter 10 μm was coupled by a gap junction of \( R_c = 10^8 \) Ω to the tip of the passive dendrite of length 600 μm of a neuron with a passive soma of surface 400π μm². Integrations of membrane and cable equations were performed using Network. In all passive compartments \( R_m = 40 \) kΩcm², \( R_i = 10^6 \) Ωcm, and \( C_m = 1 \) μF/cm² to approximate data from hippocampal basket cells (Fukuda and Kosaka 2003).

MODEL 3. Each of two cables was modeled as cylinders of length 600 μm divided into six compartments of equal length. The membrane potential of compartment \( j \) (indexed from 0 to 5) of cable \( i \) is denoted by \( V_{i,j} \). The two cables were gap-junctionally coupled at their ends, connecting segments 1.5 to 2.0. The compartments are built with specific membrane resistivity \( R_m = 40 \) kΩcm², specific axial resistivity \( R_i = 60 \) Ωcm, and specific membrane capacitance \( C_m = 10^{-6} \) F/cm² (Hartline and Castelfranco 2003). In these simulations \( V_{1,0} \) was voltage clamped to produce a sinusoidal change in voltage, \( V_{1,0}(t) = A \sin(\omega t) \), where \( A \) is a scaling factor.

**RESULTS**

An optimal diameter exists when two cables are coupled by gap junctions

Using basic equations of cable theory, it can be readily demonstrated that an elongated process becomes more electrotonically compact if its diameter increases. This is explained by the fact that the length constant of a uniform cable is proportional to the square root of its diameter. Therefore the larger the diameter, the larger the constant \( \lambda \) and the smaller the electrotonic length (\( \text{len}/\lambda \)) of the cable. Consequently, if one end of a finite sealed-end cable is voltage clamped (at \( V_{1,0} \neq V_{\text{ref}} \)), the voltage attenuation along the cable is less if the cable diameter is larger, and thus the voltage at the distal end of the cable is closer to \( V_{1,0} \).

If, additionally, this cable is coupled at its distal end by gap junctions to a second cable, there will be a voltage drop across the gap junction and the voltage attenuation will continue along the second cable. This is demonstrated in Fig. 1A by plotting the voltages along two cables, each of length \( \text{len} = 600 \) μm, coupled at the end with a gap junction of resistance \( R_c \), when \( V_{1,0} \) is voltage clamped at 40 mV. These traces were calculated using Eqs. 1 and 5 for cable 1. For cable 2, Eq. 6 was used to determine \( V_{2,0} \), Eq. 7b to determine \( V_{2,1} \), and Eq. 1 with \( V_0 = V_{2,0} \) to determine \( V_{2,\infty} \). As the coupling resistance \( R_c \) is decreased, the voltage drop across the gap junction becomes less pronounced and the two coupled cables increasingly resemble a single cable with length = \( \text{len}_1 + \text{len}_2 \) (Fig. 1A).

Figure 1B shows the voltages along the two cables as the diameters of both cables are simultaneously varied. The top four traces correspond to voltages at four equidistant positions along cable 1 and the bottom four traces to voltages at equidistant positions along cable 2 (as indicated by arrows in the schematic diagram). As expected, we found that when the beginning of cable 1 was voltage clamped, the voltage attenuation along this cable monotonically decreased as its diameter increased. This was not the case for voltages along cable 2.
Although for any fixed diameter there was voltage attenuation along cable 2 (Fig. 1A), as the diameter was increased, at any given position along cable 2 the voltage first increased and then decreased (Fig. 1B, bottom four traces). Thus for each position along cable 2, there was a cable diameter at which the voltage attenuation was minimal. We refer to this value as the optimal diameter and to the voltage versus diameter graph as a diameter tuning curve.

A qualitatively similar result was obtained in current-clamp conditions with a constant current applied to the beginning of cable 1. However, the optimal diameter obtained in current clamp was more than one order of magnitude smaller than that in voltage clamp (Fig. 1C). An optimal diameter was also observed if a fixed conductance—the equivalent of a synaptic current input \( i_{syn} = g_{syn}(V - E_{syn}) \)—was applied to the beginning of cable 1 (Fig. 1C). In this study we will analyze only the case when the beginning of cable 1 is voltage clamped. The other cases can be treated in a similar fashion.

To understand how the optimal diameter emerges, we used analytical expressions for the steady-state voltages along two uniform cables of finite length, coupled at the end with a gap junction (METHOIDS, Eqs. 1–7). We will show that the optimal diameter depends on the gap-junction resistance as well as the membrane properties of both cables.

Using Eqs. 5 and 6 we compared the voltages at the two sides of the gap junction \((V_{1,0} \text{ and } V_{2,0})\) when the proximal end of cable 1 \((V_{1,0})\) was voltage clamped (compare dotted and solid black traces in Fig. 2A). As expected, \(V_{1,0}\) (dotted trace) approached the value of \(V_{1,0} (-40 \text{ mV})\) as the cable diameters \((d)\) increased. Within a range of relatively small \(d\) values, the voltages across the gap junction were close in value and \(V_{2,0}\) (gray trace) tracked \(V_{1,0}\). However, as \(d\) increased further, \(V_{1,0}\) approached a plateau but \(V_{2,0}\) began to decrease. The rise and fall of \(V_{2,0}\) as a function of \(d\) can be readily explained using Eq. 6 and the dependency of \(R_{m,2}\) on diameter. At small \(d\) values, the input resistance of cable 2 \((R_{in})\) is relatively large. In fact, in this range of \(d\), \(R_{m,2}\) is much larger than the gap junction resistance \(R_{g}\) and thus, to a first approximation, \(R_{g}\) can be ignored in the denominator of Eq. 6 and \(V_{2,0}\) tracks \(V_{1,0}\), which is increasing. However, \(R_{m,2}\) decreases as \(d\) increases \([R_{m,2} = R_{0,2} \coth L_{2} = (2/\pi)\sqrt{R_{g}/R_{0}}\coth (2L_{2}/\sqrt{R_{g}/R_{0}})]\) and, for large \(d\), \(R_{m,2}\) becomes much smaller than \(R_{g}\). Thus for large \(d\), the value of \(V_{2,0}\) decreases with \(R_{m,2}\) even as the value of \(V_{1,0}\) continues to increase and approaches a constant (Eq. 6). In effect, \(R_{g}\) acts as a current limiter; as its diameter increases, cable 2 becomes more “leaky” and the \(R_{g}\)-limited current flowing into cable 2 results in a progressively lower current density and more attenuated voltage change along cable 2.

Note that the existence of the optimal diameter is not limited to \(V_{2,0}\) but an optimal diameter exists for all positions along cable 2 as seen in the voltage at the distal end of cable 2 from the gap junction \((V_{2,2}, \text{black trace})\) in Fig. 2A).

This current-limiting effect of the gap junction can also be demonstrated by voltage clamping cable 1 at its distal end, next to the gap junction \((V_{1,1})\) and plotting \(V_{2,0}\) as a function of diameter (Fig. 2B, gray trace). The value of \(V_{2,0}\) is close to \(V_{1,0} (-40 \text{ mV})\) for small \(d\) and drops as \(d\) increases but, in this case, there is no optimal diameter for \(V_{2,0}\). Note, however, that all portions of cable 2 except the point immediately adjacent to the gap junction (e.g., \(V_{2,2}, \text{black trace}\) in Fig. 2B) show an optimal diameter. This is explained by the fact that the term \(1/\cosh (L_{2})\) in Eq. 7b grows monotonically as the diameter of cable 2 increases, whereas \(V_{2,0}\) monotonically decreases, independent of the voltage at \(V_{1,1}\). The product of these two terms (Eq. 7b) generates the peak voltage at an optimal diameter.

Note that the drop in \(V_{2,2}\) is primarily explained by the fact that \(R_{m,2}\) decreases as the diameter of cable 2 increases. Thus
when the diameter changes are restricted only to cable 1 (Fig. 2C, dashed trace) no optimal diameter is observed. However, if only the diameter of cable 2 is modified an optimal diameter appears (Fig. 2C, dotted trace). The optimal diameter occurs at a larger value when the diameters of both cables are simultaneously modified (Fig. 2C, solid trace) because at small diameters, $V_{\text{opt}}$ is significantly more attenuated compared with $V_{\text{opt}}$, when $d_1$ is fixed, effectively “pushing” the left side of the diameter tuning curve down and the optimal diameter to the right.

Dependency of optimal diameter on membrane properties

The steady-state voltage profile along a cable depends on the specific membrane resistance $R_m$, specific axial resistance $R_a$, length, and diameter. Figure 3 shows the effects of $R_m$ and $R_a$ on the optimal diameter of two cables coupled with $R_a = 2 \times 10^3 \Omega$. The dependency of optimal diameter on $R_m$ is most pronounced when only $R_m$ is increased (Fig. 3A, open triangles), showing an initially rapid, followed by a more gradual, decrease. The effect of the $R_m$ of cable 2 on optimal diameter also shows an initially rapid (but less pronounced) decrease as $R_m$ increases up to approximately 5 kΩ·cm². At this point, and in contrast to what is seen when $R_m$ is varied, the optimal diameter starts to increase linearly with $R_m$ (open circles). The combined effect of simultaneously changing $R_m$ in both cables is a rapid decrease of the optimal diameter as $R_m$ increases up to approximately 10 kΩ·cm² and an apparent independence of the optimal diameter above this point (filled circles). Increases in voltage attenuation resulting from decreases in $R_m$ are accompanied by an increased sharpness in the diameter tuning curve (Fig. 3B).

The optimal diameter along cable 2 can be readily derived for the signal at the distal end ($V_{2_D}$) by evaluating $\frac{\partial V_{2_D}}{\partial d} = 0$ using Eq. 7b. Assuming that the two cables are identical and covary in diameter, this equation can be simplified to obtain

$$\tanh 2L = \frac{2LR}{3R_a - 4R_aL}$$

\hspace{1cm}(11)

Equation 11 can be used to approximate the dependency of the optimal diameter on the membrane parameters, $R_m$, $R_a$, and $len$. The left-hand side of Eq. 11 (i.e., $\tanh 2L$) is bound between 0 and 1. The right-hand side has a vertical asymptote when the denominator is zero. At low values of $d$ this vertical asymptote occurs close to the point of intersection with $\tanh 2L$; i.e., the value of $d$ at which $3R_a - 4R_aL = 0$ (say $\tilde{d}$) is near the optimal diameter $d^*$. Thus solving for the value of $d$ at the vertical asymptote provides a good approximation of the dependencies of $d^*$ on membrane parameters. This value, which is valid only for relatively large values of $R_m$ (in our case $\geq 10$ kΩ·cm²), is given by

$$\tilde{d} = d \sqrt{\frac{3\pi R_a}{\sqrt{R_a}}}$$

\hspace{1cm}(12)

Note that $\tilde{d}$ is independent of $R_m$, reflecting the independence of the optimal diameter from variations of $R_m$ in both cables $\geq 10$ kΩ·cm² (Fig. 3A, filled circles).

The optimal diameter changes in a monotonic fashion whether $R_1$ or $R_2$ is modified, with the optimal diameter increasing as $R_i$ increases. As in the case of $R_m$ variations, the effect is more pronounced when $R_i$ (Fig. 3C, open triangles) is modified than $R_0$ (open circles). This effect is even more pronounced when $R_i$ values in both cables are simultaneously changed (filled circles). Equation 12 indeed shows that, for two

![Effect of membrane properties on optimal diameter. Analytical results calculated for 2 cables of equal length (600 µm) connected by a gap junction of $R_g = 2 \times 10^3 \Omega$. Beginning of cable 1 was clamped to 40 mV and diameters of both cables were varied simultaneously. A: optimal diameter for different $R_m$ values of only one cable (open circles; $R_m = 40$ kΩ·cm²), only cable 2 (open circles; $R_m = 40$ kΩ·cm²), or both cables (filled circles). $R_a = 60$ Ωcm. B: steady-state diameter tuning curve measured at end of cable 2 (both cable diameters varied simultaneously) for $R_m$ values indicated by arrows in $A$ (40 and 10 kΩ·cm²) to show effect on attenuation (top) and sharpness (bottom; voltages normalized to curve maxima for enhanced visibility) of tuning curve. C: optimal diameter for different $R_i$ values of only one cable (open circles; $R_i = 60$ µm), only cable 2 (open circles; $R_i = 60$ µm), or both cables (filled circles). $R_m = 40$ kΩ·cm². D: steady-state diameter tuning curve measured at end of cable 2 (both cable diameters varied simultaneously) for $R_i$ values indicated by arrows in $C$ (60 and 160 µm) to show effect on attenuation and sharpness of tuning curve.

\hspace{1cm} J Neurophysiol • VOL 95 • JUNE 2006 • www.jn.org
identical cables, the optimal diameter variations are proportional to $\sqrt{R}$. The effect of $R_i$ on optimal diameter is somewhat comparable to the effect of $R_m$ to the degree that changes that increase voltage attenuation along either of the two cables (i.e., by reduction of $R_m < 10 \, \text{k} \, \Omega \, \text{cm}^2$ or increase of $R_i$) increase the optimal diameter value. As in the case of $R_m$ changes, increases in voltage attenuation arising from changes in $R_i$ are accompanied by an increased sharpness of the diameter tuning curve (Fig. 3D).

Changes in cable length have a similar effect as changes that increase voltage attenuation (i.e., decreased $R_m$ or increased $R_i$). Figure 4A shows that increasing the length of cable 1 (open triangles) leads to an almost linear increase in optimal diameter. Increasing the length of cable 2 leads to an initial reduction of optimal diameter at low length values and then an almost linear increase for higher values (Fig. 4A, open circles). When both cable lengths are increased simultaneously, the optimal diameter increases monotonically in a manner similar to how optimal diameter varies with $\ell$. Increasing the length of cable 2 leads to a marked shift in the optimal diameter value. As in the case of $R_m$ increases in voltage attenuation arising from changes in $\ell$ increases monotonically in a manner similar to how optimal diameter varies with $\ell$. Increasing the length of cable 2 leads to an almost linear increase in optimal diameter at low length values and then an almost linear increase for higher values (Fig. 4A, open circles).

Dependency of optimal diameter on gap junction resistance

Gap junction resistance sensitively determines optimal diameter for signal transfer between coupled cables. As gap-junction resistance increases, the optimal diameter sharply decreases (Fig. 5A). Notice that a drop of nearly 70% occurs between gap-junction conductances of $10^6$ to $10^7 \, \Omega$ (Fig. 5A). However, whereas at $R_c = 10^9 \, \Omega$ the optimal diameter is reduced by over one order of magnitude compared with $R_c = 10^7 \, \Omega$ (Fig. 5A), the amplitude of the signal is attenuated only about 50% (Fig. 5B, top). As in the case of $R_m$, $R_c$ and cable length, Eq. 12 confirms that optimal diameter is proportional to $1/\sqrt{R}$ when both cables are identical (Fig. 5A). As described before for the effects of $R_m$, $R_c$, and length on optimal diameter,

\begin{equation}
R_i = \frac{1}{2} R_m \left( \frac{\ell_1}{\ell_2} \right) + R_c
\end{equation}

For the case when both cables are identical (Fig. 5D), $R_i = 0$, and Eq. 12 demonstrates a simple relationship of the optimal diameter to $R_c$ and cable length $\ell$.

\begin{equation}
R_m = \frac{1}{2} R_c \left( \frac{\ell_1}{\ell_2} \right)
\end{equation}

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attenuation of the signal is accompanied by an increased sharpness in the diameter tuning curve (Fig. 5B, bottom).

Effect of branching on optimal diameter

Dependency of the optimal diameter on membrane properties suggests that different branches in a dendritic tree, having different membrane properties, may also express different optimal diameters. Indeed, Fig. 6A shows that dendrites with different membrane resistances but otherwise identical properties display different optimal diameters. When the diameters of both the mother cable 2 and its daughter branch (branch 1) are varied simultaneously the cable with lower input resistance (Fig. 6A, red trace) shows a larger optimal diameter and a sharper but more attenuated diameter tuning curve. This effect is local because diameter changes in a daughter branch produce an optimal diameter only in that branch (Fig. 6B, red trace). Diameter changes in a mother dendrite (cable 2) will produce almost the same optimal diameter in that dendrite and its daughter branches, with exact values depending on the specific membrane properties of the daughter branch (Fig. 6C; note that the postjunctional segment closest to the gap junction is part of the mother branch; see schematic diagram).

Series of cables connected end to end

Cables of similar properties connected end to end in series (see diagram in Fig. 7A) correspond to a condition analogous to that found in some biological tissues, most notably cardiac muscle (Joyner et al. 1984) and some striated muscle fibers (e.g., in Drosophila; G. Davis, personal communication). Figure 7A shows the response, measured at the distal ends ($V_{l,n}$), of three identical cables connected end to end when the proximal end of cable 1 ($V_{l,0}$) is voltage clamped to 40 mV (see schematic diagram). As in the case of two cables connected in a similar manner, the end of cable 2 shows a clear optimal diameter for signal transfer. However, in contrast with
All cables were built with traces were varied. In red the diameter of both cables 1 and 3 (black traces) were varied or only cable 3 (dashed trace), only cable 2 (dotted trace), and connected at one end by a gap junction of resistance \( R = 40 \, \text{kOhm cm} \), and \( R_{i} = 60 \, \Omega \), and \( \text{len} = 600 \, \mu\text{m} \) and connected at one end by a gap junction of resistance \( R_{g} = 10^{6} \, \Omega \). Voltage at the beginning of cable 1 was clamped to 40 mV. A, top: schematic diagram of connectivity. Bottom: \( V_{1, L} \) (solid line), \( V_{2, L} \) (dashed line), and \( V_{3, L} \) (dotted line) shown as a function of cable diameter when the diameters of all 3 cables were varied simultaneously. B: \( V_{1, L} \) shown as a function of cable diameter when the diameters of only cable 1 (solid trace), only cable 2 (dashed trace), or only cable 3 (dotted trace) were varied. C: \( V_{1, L}, V_{2, L}, \) and \( V_{3, L} \) shown when the diameter of both cables 1 and 3 (black traces) or both cables 2 and 3 (red traces) were varied. In B and C, the fixed-diameter cables had diameter = 10 \( \mu\text{m} \).

In the two-cable case, the three-cable case shows a slightly smaller optimal diameter when measured at the end of cable 2 (2.5 \( \mu\text{m} \) compared with slightly >3 \( \mu\text{m} \) in the two-cable case; compare dashed trace of Fig. 7A with bottommost trace in Fig. 1B). Furthermore, the end of cable 3 shows a still lower optimal diameter value, a sharper tuning curve, and a more attenuated amplitude at all cable diameter values (Fig. 7A, bottom trace). Figure 7B shows that variation in diameter of the individual cables has very different effects on both the attenuation of signals and the optimal diameter value. As in the two-cable case, varying the diameter of cable 1 alone affects exclusively the attenuation of the signal measured at the end of the last cable and there is no optimal diameter (Fig. 7B, solid trace). When only cable 2 diameter is varied, a sharp peak (optimal diameter) appears in the cable diameter tuning curve (Fig. 7B, dashed trace), whereas variations in cable 3 diameter alone produce a less attenuated and broader cable diameter tuning curve with a significantly lower optimal diameter value (Fig. 7B, dotted trace). A similar result is observed if the diameters of all but one of the three cables are varied simultaneously: as the cables whose diameters are varied are placed farther away, the optimal diameters values become shorter and the amplitudes less attenuated (Fig. 7C). In other words, middle cable diameters in end-to-end connection configuration have a stronger effect on signal attenuation but a weaker effect on tuning of the diameter tuning curve.

Cables connected in a middle position

Dendro-dendritic gap junctions are common in the CNS (Fukuda and Kosaka 2003; Matsumoto et al. 1988; Sotelo et al. 1986). Also, recent studies have proposed that axo-axonic gap junctions are present between hippocampal pyramidal neurons and account for the generation of spikelets (Schmitt et al. 2001; Traub et al. 1999, 2003). Results of the current study suggest that transmission of signals between two axons, between axons and other neuronal processes, or between dendrites depends on the diameter of the processes involved. We addressed this hypothesis by examining the effects of gap-junctional coupling between two cables at intermediate positions along the cable (see schematic diagrams in Fig. 8). We considered two different configurations: coupling between sealed cables (Fig. 8, A and B) and coupling between semiinfinite cables (Fig. 8, C and D; see METHODS). As before, in both cases we looked at the steady-state voltages along cable 2 when the beginning of cable 1 was voltage clamped. Under these conditions our results qualitatively correspond to those observed for end-to-end coupled cables: in both cases an optimal diameter for signal transmission is present at all positions along cable 2 (e.g., dotted and dashed traces in Fig. 8A for the sealed-end case, and Fig. 8C for the semiinfinite case), whereas no optimal diameter is observed along cable 1 (solid traces in Figs. 8, A and C). The main differences between these two cases are that the sealed-end cable shows less signal attenuation across the gap junction and larger optimal diameter values than those of the semiinfinite cables. In both cases, the optimal diameter is dependent on the gap junction position. As the gap junction is moved farther away from the sealed end, both the optimal diameter and signal attenuation increase (Fig. 8, B and D).

Gap-junction–mediated coupling potentials

To further examine the effects of diameter on a signal transmission analogous to axo-axonal coupling, we coupled an action potential–generating single-compartment model neuron to the center of a long multicompartment cable (Model 1 in METHODS; Fig. 9, schematic diagram). We found that the cou-
The coupling potential produced by the action potential in the coupled cable shows a maximal amplitude at a unique cable diameter, in this case at about 5 \( \mu \text{m} \) (Fig. 9, top inset). The coupling potential amplitude was diminished if the axon diameter was different. One interesting consequence of the optimal diameter is that, although the amplitude of the coupling potential produced at a small diameter (e.g., 1 \( \mu \text{m} \), leftmost inset in Fig. 9) may be almost identical to the amplitude at a diameter higher than the optimal value (e.g., 30 \( \mu \text{m} \), rightmost inset in Fig. 9), the time course of these coupling potentials varies substantially because of the different time constants of the membrane at these different diameters.

The functional implications of the existence of an optimal diameter for signal transmission across a gap junction are further illustrated in the multicompartment model of dendro-dendritic propagation of an action potential (Model 2 in METHODS; Fig. 10A). Figure 10A shows an action potential propagating from the axon, through a passive soma and into the tips of passive, uncoupled dendrites of different diameters. The membrane potential amplitude at the tip of the dendrites increased with the diameter of the dendrite and no optimal diameter was observed. In contrast, when the tip of one of the dendrites was electrically coupled to the tips of the dendrite of a second (ball-and-stick) passive neuron, the resulting electrical coupling potential recorded at the soma of cell 2 had maximal amplitude for the dendrite of diameter 5 \( \mu \text{m} \) (Fig. 10B). The results shown in Fig. 10B imply, for example, that only the postsynaptic neurons with dendrite diameters of 2 and 5 \( \mu \text{m} \) would produce action potentials in response to the electrical coupling potential, if the spike threshold is about 8 mV above the resting potential.

**Effects of signal frequency on optimal diameter**

Although we do not present here the analytical solutions to the transient (non-steady-state) case, the behavior of two end-to-end coupled cables in response to a sinusoidal change in voltage at the beginning of cable 1 can be intuitively understood in a way similar to the effects of \( R_m \) on voltage attenu-
DISCUSSION

Recent experimental results have shown gap junctions to be much more prevalent in the nervous system than previously known. Few pharmacological agents affect gap junctions specifically and direct experimental measurement or manipulation of electrical coupling is notoriously difficult (for a review see Connors and Long 2004). Consequently, theoretical studies of electrical coupling are necessary to understand the network consequences of its interactions with the membrane properties of the coupled cells. We examined the interaction between electrical coupling and cable properties of coupled processes on signal transfer attenuation. We show that two gap-junctionally coupled passive cables will produce maximal signal transfer (coupling potential) at a certain “optimal” diameter. Such an optimal diameter exists for both steady-state signals and action potentials or periodic signals and occurs at similar values for frequencies up to \( \approx 200 \) Hz. This optimal signal transfer may be potentially important in the operation of neuronal systems that involve gap-junctional communication, where coupling potentials of optimal amplitude may result in activating regenerative events such as action potentials, plateau potentials, and voltage-dependent membrane potential oscillations (Fig. 10).

**FIG. 10.** Effect of cable diameter on action potential propagation through passive dendrites and across a gap junction. A: membrane potentials of an invading action potential in the soma (\( V_{1,\text{soma}} \)) and at the tips of 6 dendrites (\( V_{1,\text{dendrite tip}} \)) of varying diameters are shown. Neuron was built with a 6-compartment spiking axon, a passive soma, and 6 passive, 600-\( \mu \)m-long dendrites (made of 6 compartments, each 100 \( \mu \)m long), \( R_m = 40 \) k\( \Omega \)cm\(^2\), and \( C_m = 1 \) \( \mu \)F/cm\(^2\) (schematic diagram). B: distal tip of the 10-\( \mu \)m-diameter dendrite of the same neuron as in A (Cell 1) was coupled to the tip of the “dendrite” of a ball-and-stick passive neuron (Cell 2; schematic diagram). Diameter of the dendrite of Cell 2 was varied and the PSP recorded at the soma of Cell 2. Cell 1 was coupled to only one Cell 2 in each simulation run (black resistor symbols in schematic diagram) and the diameter of Cell 2 was changed in each run (gray resistor symbols). \( R_c = 10^7 \) \( \Omega \). Horizontal scale bars are 10 ms long.

![Diagram A](image1)

![Diagram B](image2)

atation and the presence of an optimal diameter for signal transfer along coupled cables. We used numerical simulations of compartmentalized cables (Model 3 in **METHODS**) to show that as the frequency of the input signal (\( V_{1,0} \)) increases and the impedances of the cables decrease, the amplitude of the output signal decreases (\( V_{2,1} \); Fig. 11A). At the same time, the optimal diameter gradually increases (stars and vertical traces) very much like the optimal diameter increases as \( R_m \) (of both cables simultaneously) decreases in the steady-state case (Fig. 3A, solid symbols). The increased signal attenuation at high frequencies is also accompanied by a sharper diameter tuning curve (Fig. 11B) similar to that observed in the steady-state case (Fig. 3B).

**FIG. 11.** Effect of input frequency on optimal diameter. Two cables of length = 600 \( \mu \)m (6 compartments of equal length each), \( R_m = 40 \) k\( \Omega \)cm\(^2\) and \( R_c = 60 \) \( \Omega \)cm, were coupled with a gap junction of resistance \( R_g = 2 \times 10^7 \) \( \Omega \). Sinusoidal voltage-clamp signals of amplitude 20 mV with different frequencies were applied to the first compartment of cable 1 and the diameters of both cables were varied simultaneously. A: graph shows diameter tuning curves at the distal end of cable 2. Range of frequencies from top to bottom: 0–1,000 Hz. Some frequencies are indicated for reference. Stars mark the optimal diameter at each frequency. Lines connecting the stars are added for increased visibility. B: diameter tuning curves for 0 Hz (pulse), 20, 50, and 200 Hz, normalized to their peak values, showing the increased sharpness of the curves, but little change in optimal diameter, with increasing input frequency.
Comparison with optimal diameter of a single cable in current clamp

A previous study showed that a single cable has an optimal diameter at the distal end when a constant current or conductance is injected in the proximal end (Holmes 1989). This single-cable optimal diameter is determined exclusively by the electrotonic length $L$ of the cable, is independent of the input resistance and the terminating resistance, and, furthermore, it disappears when the cable is voltage clamped. The optimal diameter of two coupled cables, described in this study, is qualitatively distinct from that of the single cable described by Holmes (1989) and cannot be reduced to that case. In particular, the optimal diameter in our study is dependent on the input resistances and length constants of the coupled cables as well as the gap junction resistance (Eq. 11) and, moreover, it occurs in voltage-clamp (this study) as well as in current-clamp conditions and conductance-clamp conditions (Fig. 1C). Additionally, the current-clamp optimal diameter described by Holmes (1989) occurs at values an order of magnitude smaller than the optimal diameter of two gap-junctionally coupled cables. Specific membrane resistance ($R_m$) values measured in vertebrate neurons using the whole cell patch-clamp technique are 10–70 kΩ cm$^2$ (see references in Coleman and Miller 1989). For invertebrates, the values are commonly lower (Rall 1977) but can also be in the higher range (Hartline and Castelfranco 2003). For such $R_m$ values, the current-clamp optimal diameter in single cables is below physiologically realistic levels (<0.1 μm; Berthold 1978; Mikelberg et al. 1989) but the optimal diameters for gap-junctionally coupled cables fall within the physiological range (0.1–10 μm; Figs. 3–5) in either current-, conductance-, or voltage-clamp conditions (Fig. 1C).

Relationship between optimal diameter and cable properties

It may seem that the optimal diameter arises because, with increasing diameters, there is better electrotonic access to the distal points but that larger diameters put a larger load on the current source. These two opposing effects would thus produce optimality. However, although this intuition is correct for the current-clamp single-cable case, it is insufficient in general because these opposing effects also occur in a single voltage-clamped cable without producing an optimal diameter. In the case of two coupled cables, the existence of the optimal diameter depends crucially on the limiting effect of the gap junction on current flow. The gap junction acts as a voltage divider that limits the current flow into the second coupled cable. Therefore although the signal at the end of cable 1 monotonically approaches the voltage of the proximal end, the current-limiting effect of the gap junction forces the signal along cable 2 to decay with diameter past a certain value, thus generating an optimal diameter for signal transmission.

A necessary requirement for an optimal diameter to appear is that at least one of the two coupled cells has a cablelike structure (Fig. 4). Two isopotential coupled cells do not exhibit an optimal diameter. A further general rule is that any parameter changes that result in an attenuation of the voltage signal along the coupled cables results in a sharpening of the diameter tuning curve along the second cable (Figs. 3–5).

Action potentials effectively voltage clamp the membrane to the action potential waveform. Thus the occurrence of an action potential in a neuron presynaptic to the gap junction will produce a maximal coupling potential for a unique optimal diameter. This is shown in a simplified configuration in Figs. 9 and 10. Note that an action potential does not produce an optimal diameter in a single cable (Fig. 10A). Furthermore, a strong chemical synapse that produces a strong conductance change is effectively equivalent to a voltage-clamp input and will produce an optimal diameter in processes that are gap-junctionally coupled to the cell receiving the synaptic input. However, such input does not produce an optimal diameter in a single cable (Holmes 1989).

A further effect of the sensitivity of electrical coupling to cable diameter is that a signal (such as an action potential) may be transmitted with identical attenuation into cables of different diameters if their diameters lie at either side of the optimal value, for instance at 1 and 30 μm in Fig. 9. However, the synaptic integration properties of these two cables can be substantially different because of the different membrane properties of cables of different diameter (see coupling potential shapes in Fig. 9 at 1 and 30 μm, and Fig. 10B). Additionally, the appearance of the optimal diameter is not restricted to a pair of coupled cables, to cables coupled only at their ends, or to sealed cables. Fine sealed (Fig. 8, A and B) or very long cables coupled in middle positions (Fig. 8, C and D), as is the case with axo-axonal (Schmitz et al. 2001; Yasargil and Sandri 1990) and dendro-dendritic gap junctions (Fukuda and Kosaka 2003; Matsumoto et al. 1988; Sotelo et al. 1986), respectively, as well as open-ended cables (not shown), all exhibit optimal diameters. Moreover, an architecture of series of cables coupled end to end, which may be considered equivalent to some types of muscle cells such as cardiac myocytes (Joyner et al. 1984), also demonstrates an optimal diameter.

Another important property of the optimal diameter is that, in a branched postjunctional structure, it is local to the daughter branch (Fig. 6). Thus a mother cable and its daughter branches may be tuned to have distinct diameters near their optimal values depending on their different membrane properties. This is potentially important because it may allow for functional compartmentalization based purely on this geometrical condition.

Diameter and gap junction conductance measurements

Optimal signal transfer by gap junctions is a local effect (Fig. 6). Thus any direct experimental test of such optimal signaling requires measurement of gap-junction conductances specific to the coupled processes. Few such simultaneous measurements have been performed (Fukuda and Kosaka 2003). We predict the optimal diameter value for normally observed gap-junction conductances specific to the coupled processes. Few such simultaneous measurements have been performed (Fukuda and Kosaka 2003; Pappas and Bennett 1966; Tamas et al. 2000). Using Eqs. 7, 9, and 10 for cables coupled along middle positions (see METHODS) we estimated the optimal dendrite diameter of hippocampal basket cells with dendrite length and gap junction position values reported by Fukuda and Kosaka (2003), $R_m$ and $R$ values reported by Saraga et al. (2003), and $R_\text{s}$ values reported by Fukuda and Kosaka (2003) and Traub et al. (2001). Average membrane parameter values
Dendritic pruning during critical stages of development is important in the establishment of functional networks and is known to rely on the strengthening of correlated signals between cells (Hata et al. 1999; Kandler and Katz 1998). Neuronal structure and circuit formation during these critical periods rely on chemical and electrical coupling (Kandler and Katz 1998) and dendrite morphology (branching, length, spine density) is regulated by activity (Konur and Ghosh 2005). Gap-junctionally coupled processes that are most strongly coupled (e.g., at an optimal diameter) are thus likely to be selected and preserved during pruning. Consequently, it is conceivable that cable diameter, like other morphological neuronal features, may also be regulated during development (Konur and Ghosh 2005) and thus be another important variable in the determination of network structure and activity.

Network synchronization

Gap-junctional coupling among interneurons is important in the generation of synchronous activity in different regions of the mammalian brain (Connors and Long 2004). These interneuron networks involve co-localized chemical and electrical coupling (Beierlein et al. 2000; Friedman and Strowbridge 2003; Tamas et al. 2000; Traub et al. 2001) both of which may be involved in producing synchrony (Chow and Kopell 2000; Kopell and Ermentrout 2004; Lewis and Rinzel 2003). We have shown that optimal diameter depends on the membrane resistance of the coupled processes, particularly when this resistance is low (Fig. 3A). Therefore when chemical synaptic input is low (high $R_{in}$), pairs of neurons could be tuned to be maximally coupled by gap junctions allowing for effective synchronization. Such a mechanism may be at work where synchrony appears after blocking synaptic transmission (Angstadt and Friesen 1991). Alternatively, synchronization can be driven by chemical synaptic inputs (low $R_{in}$), bringing the electrical coupling-based signaling out of optimal range. In this way synchrony could be ensured by different cellular mechanisms. Moreover, electrical coupling and synaptic inputs can act synergistically to bring about synchrony (Friedman and Strowbridge 2003; Kopell and Ermentrout 2004; Tamas et al. 2000).

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