Slow Temporal Filtering May Largely Explain the Transformation of Stick Insect (Carausius morosus) Extensor Motor Neuron Activity Into Muscle Movement

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INTRODUCTION

Muscles often transform motor neuron activity into contraction in complex manners (for examples, see Aplysia feeding system [Brezina et al. 2000; Zhurov and Brezina 2006], crustacean stomatogastric system [Morris and Hooper 1997, 1998, 2001; Morris et al. 2000; Thuma et al. 2003], human [Lee et al. 1999a,b; Thomas et al. 1999], rat [Abbate et al. 2000; Grottel and Celichowski 1999; Van Luteren and Sankey 2000], stick insect [Hooper et al. 2006a,b]). Understanding the neuro muscular transform is important because a major goal of neuroscience is understanding the neural basis of movement generation, and this goal can be achieved without understanding how muscles respond to their driving input. For example, crustacean stomatogastric networks produce a wide range of motor neuron output patterns (for references, see Selverston 1995). It is often assumed that these patterns would produce different behaviors. However, muscle response to neuron input could be sufficiently nonlinear that large changes in neuron activity produce little change in muscle output. The functional relevance of the stomatogastric system’s many output patterns can thus not be understood without knowing how its muscles respond to neural input.

Similar interpretation difficulties are expected in any system in which muscles have complex responses to motor neuron input. The stick insect has been extensively used to study central and peripheral mechanisms underlying locomotion (for references, see Büschges 2005). Tonic stimulation paradigms have recently been used to describe stick insect muscle biophysical properties (Guschlbauer et al. 2007), and we have in parallel been examining the response of these muscles to stimulation with spike patterns observed during single middle leg walking. We showed earlier that, in these preparations, a wide variety of motor neuron firing patterns occur, and these patterns induce a wide variety of contractions in isolated extensor muscles (Hooper et al. 2006b). This contraction variability likely arises (see results) not from an intrinsic randomness in muscle response, but instead from the large variability present in motor neuron activity, i.e., muscle contraction in this species is likely a deterministic function of motor neuron input. This wide variability of physiologically relevant motor neuron activity patterns and muscle responses makes the stick insect neuromuscular system an excellent preparation for studying how neuron input is transformed into movement. We have also shown that spike number, not frequency, primarily codes for amplitude during contraction rises in these muscles (Hooper et al. 2006a), which implies that these muscles are slow filters on the time scales of the interspike intervals (5–10 ms) present in early parts of extensor motor neuron bursts.

We have, however, reported how extensor muscle intrinsic properties affect muscle response over the much longer times scales (hundreds of milliseconds) present in complete extensor motor neuron bursts. Muscle response to spiking variation over long time scales was particularly interesting because substantial (up to 50%), long-lasting (hundreds of milliseconds) spike frequency declines often occurred in these bursts without any corresponding decrease occurring in muscle

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contraction amplitude. We were at a loss to explain this difference. Simultaneously with the experimental work, we were studying methods to obtain a mean extensor spike frequency burst profile and using these profiles to drive simple low-pass filter models. In working with these models, we found to our surprise that certain combinations of linear input patterns, when filtered by models with particular time constants, could transform declining inputs into sustained outputs. We studied this process in some detail and show here that it results from a dynamic interaction between the existing value of the model output and the constantly shifting steady-state value to which the model is driving as the input declines. We therefore quantified extensor muscle temporal properties to test whether a more realistic, but still completely slow filter based, model could reproduce real muscle contractions.

Two interesting results from this work are that the muscles have different time constants, rising much more slowly than they relax, and both time constants, even in individual muscles, vary across wide ranges. The rise time constant is extremely slow (200–700 ms), and we show here that slow temporal filtering using these data can reproduce the experimental data, although to obtain best fits it is necessary to allow the time constants to vary (but always within the physiological range) from contraction to contraction. We also examined the effect of varying model time constant across the observed range and show that these variations can induce substantial changes in output shape (for the same pattern of motor neuron input), particularly at the extremes of the observed time constant values. These data suggest that, as in the Aplysia feeding system, time constant variation could be used to alter muscle response in this system and highlight the need to understand the basis of this variation. More generally, these data emphasize how strongly muscle dynamics can transform neural input and the central importance of the neuromuscular transform in movement generation.

METHODS

All experiments were performed at room temperature on adult Carausius morosus females from a colony maintained at the University of Cologne. All animals were approximately the same size and showed vigorous responses to handling.

Neural recordings during treadmill walking

For a complete description, see Gabriel et al. 2003. In brief, all legs except the right middle were amputated at mid-coxa, and the animal was attached to a flat platform dorsal side up. Middle leg retraction and protraction were prevented with dental cement (Protemp II, ESPE). Platform height was adjusted until, at mid-stance, femur-to-tibia joint angle and the tibia-to-treadwheel angle were 90°. Gut, fat, and connective tissue were removed from the thorax. A monopolar hook electrode was used to record from nerve n3 (Marquardt 1940). Data were digitized with a Micro1401 and transferred to a personal computer using SPIKE2 software (both from Cambridge Electronic Design). Throughout the experiment, the thorax was filled with C. morosus saline (in mM: 178.54 NaCl, 10 HEPES, 7.51 CaCl₂ · 2 H₂O, 17.61 KCl, 25 MgCl₂ · 6 H₂O, pH 7.2) (Weidler and Diecke 1969). Walking was induced by tactile stimulation of the antennae or abdomen. Six walking sequences from five animals were used in this work. We were studying methods to obtain a mean extensor spike frequency burst profile and using these profiles to drive simple low-pass filter models. In working with these models, we found to our surprise that certain combinations of linear input patterns, when filtered by models with particular time constants, could transform declining inputs into sustained outputs. We studied this process in some detail and show here that it results from a dynamic interaction between the existing value of the model output and the constantly shifting steady-state value to which the model is driving as the input declines. We therefore quantified extensor muscle temporal properties to test whether a more realistic, but still completely slow filter based, model could reproduce real muscle contractions.

Second, in the work using physiological spike trains, all contractions of all seven muscles similarly varied in response to the variations in burst input present in the walking sequences, which indicates either that in all experiments the same axons were being stimulated, or if different axons were being stimulated in different experiments, this change had very little or no effect on muscle output (Hooper et al. 2006b). This lack of effect of co-activating SETi and or CI, is likely caused by the strong FETi activity present in our walking sequences; prior work showing that CI had a strong effect on muscle output (Bässler and Stein 1996) was performed with experimental paradigms that activate SETi much more strongly than FETi (Büschges 1995) and that induced much smaller muscle contractions (Bässler and Stein 1996) than those used here or in Guschlbauer et al. 2007 or Hooper et al. 2006a,b. Consistent with this observation is the fact that the muscle contains many singly innervated FETi fibers, but all SETi innervated fibers are also innervated by CI, (Bässler et al. 1996).

Muscle contraction recording

The preparation was identical to the single leg walking preparation except all leg movement was prevented with dental cement, an insect pin was used to fix the tibia to the side of the platform, and the extensor muscle tendon was severed below the femur-tibia joint and attached to an Aurora Scientific (ASI) Dual Mode Lever System. The ASI is a combined movement and force transducer in which the experimenter sets muscle rest length and a maximum force the system delivers to the muscle. At muscle forces below this level (e.g., at the beginning and end of a contraction), the ASI delivers only sufficient force to prevent muscle shortening; during these periods, the ASI thus functions as a standard isometric force transducer. At muscle forces greater than the set maximum force, the ASI delivers the set maximum force and the muscle shortens against this constant load; during these periods, the ASI thus functions as a standard isotonic movement transducer. Muscle rest length was set to the length of the muscle when the femur-tibia joint was at 90° (measured during the dissection before the extensor muscle tendon was severed). The maximum force level was always set at the minimum necessary for the muscles to relax to rest length between the contractions induced by the nerve recordings made in the single leg walking preparations. This force level is sufficiently high that the ends of the relaxations are “cut-off” by the transition of the ASI to the isotonic mode, which causes considerable difficulty in measuring relaxation time constants. However, this difficulty cannot be avoided, because otherwise, the movements temporally summate, which does not occur in real walk-
ing movements, in which the leg flexes—and hence, as here, the extensor muscle lengths—to at least the rest position with each step. Note that this same difficulty would also occur using simpler isotonic transducers, in which to achieve complete relaxations within the physiological interburst intervals, the muscle would need to be loaded with a weight sufficient that, if allowed to relax completely, it would relax beyond its rest length, which would need to be prevented by a stop being placed under the lever arm to prevent muscle overstretching (see Morris and Hooper 1997, 1998; Morris et al. 2000). Note that this is not as unphysiological as it might initially appear, because in the animal extensor lengthening likely stops not because the extensor has reached an equilibrium length against a counterforce, but instead because the limb has maximally flexed. Note also that the details of the muscle loading in no way affect the conclusions presented here because the muscle time constants and the physiologically driven contractions were both obtained with identical loading. As such, the important issue is not how muscle time constant and contraction shape would change with changes in loading, but whether the time constants measured the loading condition used correctly predict the contractions produced under the same loading.

Muscle contractions and nerve stimulations were simultaneously recorded with SPIKE2. Seven extensor muscles (each from a different animal, and all from different animals than those the walking sequences were recorded from) were used in this work. In one walking sequence, muscle fatigue occurred in most muscles (see Hooper et al. 2006a). Contractions from this sequence were not used in the work reported here.

**Subsequent analysis**

Data were analyzed and plots generated using Kaleidograph (Synergy Software) built-in functions or macros written by the authors. Figures were prepared in Canvas (ACD Systems). All error bars are SD; all averages in the text are means ± SD. In general, data analyses were standard, but two aspects of this work require additional explanation.

First, as noted above, ASI maximum force levels were set such that the muscles returned to rest length within the natural interburst intervals. These levels were sufficiently high that the low-amplitude portions of the muscle lengthenings were missing because the ASI switched to isometric mode before the muscles reached these lengths. Figure 1A shows this problem for four relaxations from one muscle. In each relaxation, the muscle returned to rest with an abrupt slope transition at the rest (0) length, as opposed to the smoothly decreasing slope expected for an exponential decline. This difficulty is highlighted when exponential fits (dashed lines) were made to the data. The fits do not well match the data and have poor R² values.

This difficulty presumably arose from the ASI switching to isometric mode at muscle lengths where length was still rapidly changing. That is, if it had been possible to set the maximum force level lower, more of the low velocity portions of the curves would have been present than is the case in Fig. 1A. This difficulty can be overcome by not using the muscle length at the set maximum force as rest length in the curve fits, but instead allowing the fitting procedure to find a shorter “rest” length to which the muscle would have relaxed if the isometric switch had not occurred. This was accomplished by fitting the data with Amplitude = rest length + Ampr · e⁻ⁿᵗ (where n is time and r is time constant), with rest length being an unfixed parameter (Fig. 1B). When this was done, rest length became negative (as expected, because the hypothesis was that the muscle did not relax to the length it would have at this load level because of the ASI switching to isometric mode), and the fits became very good.

However, this procedure introduced a new difficulty in that each fit found a different rest length, whereas the muscle presumably would have always relaxed to the same length, because in all cases, muscle load was the same. This length is unknown, but a good first approximation is presumably the mean of the rest lengths found in the fits to all the relaxations (solid horizontal line, Fig. 1B; note that this is the mean of all the relaxations of this muscle and not just of the 4 shown). When this value was used for rest length and the fits rerun (Fig. 1C), R² values only slightly decreased. Relaxation time constants were therefore calculated by this two-step method in which for each muscle fits were first made to all relaxations using Amplitude = rest length + Ampr · e⁻ⁿᵗ with rest length an unfixed parameter, the mean of the rest lengths determined in these fits was calculated, and the fits were rerun using the above equation with this mean rest length as a fixed factor. Rise time constants were also calculated using this mean rest length as a fixed factor.

A second difficulty was determining an unambiguous method for averaging motor neuron spike frequency profiles. Figure 2A shows a motor neuron burst (vertical blue and red lines at bottom of plot), the muscle shortening induced by the burst (thin black smooth line), and the instantaneous spike frequency (1/interspike interval) profile (jagged blue and red lines) of the burst. Although there is considerable variation in the spike frequency profile, it could be summarized to a first approximation by linear fits to its rising and falling portions. The difficulty with this approach is finding an unambiguous method to define the dividing time between the rising and falling portions of the profile. We resolved this issue by fitting the spike frequency profiles with cubic polynomials (thick solid black line), which can have different instantaneous slopes on either side of their inflections (note that the portion of the fit left of the vertical dashed line has a steeper slope than the portion to the right of the line). We were thus able to capture the fact that the spike frequency profiles could have rising and falling portions with different slopes. We used the uppermost inflections of the cubic polynomials to define the dividing times (vertical black dashed line) between the rising and falling portions of the spike frequency profiles and performed linear fits (straight blue and red lines) to each portion.

A third difficulty with analyzing these profiles was that some profiles had only a falling phase (Fig. 2B). Although in Fig. 2B, it is obvious that there is no rising phase, for other contractions this was not the case (does one low point at the beginning of a profile give the profile a rising phase?). The cubic polynomial fitting procedure resolved this question in that for all cases in which it was obvious there was no rising phase, the uppermost inflection of the polynomials occurred before the first spike. This criterion was therefore used to classify ambiguous spike frequency profiles as follows. If the cubic polynomial uppermost inflection occurred before the first spike, no rising phase was assumed to exist, and a (of necessity, falling) linear fit to all the data was performed (this was the case with 20—11%—of the 178 motor neuron bursts). In these cases, the dividing line between rising and falling phases was set to the burst beginning time. If the cubic polynomial’s uppermost inflection occurred after the first spike, a rising phase was assumed to exist, and the boundary between the rising and falling phases was defined as above, and linear fits to each portion were performed. A final difficulty with these spike frequency analyses was that linear fits to 51% of the rising phases (80 of 158 contractions), and 4% of the falling phases, were not statistically significant. Slope data from these fits were not used in calculating the mean spike frequency profile.

**RESULTS**

The bottom plots in Fig. 3, A–E, show the spike trains (vertical lines at bottoms of plots), spike frequencies (jagged lines), and a five-point smoothed spike frequency profile (red thick lines) for five extensor motor neurons bursts recorded during single leg walking. The top plots in each panel show the contractions (colored lines) induced by the panel’s burst in the seven extensor muscles used in this work. One important point to make about these contractions is that, although there is
considerable muscle to muscle variation in contraction amplitude, large scale contraction shape is largely conserved across muscles. For instance, all the contractions in Fig. 3A smoothly rise with a continually decreasing slope throughout most of the contraction, and show, if any, only relatively brief periods in which contraction amplitude is constant. In Fig. 3, B–D, alternatively, almost all contractions (the only exception being muscle 5, purple, in B) show long periods during which amplitude is constant. The contractions in Fig. 3E show somewhat more variation, in that muscles 3 (orange) and 6 (green) show an initial long period of almost constant contraction amplitude followed by a late rise to a sustained peak, whereas muscles 1 (black), 4 (yellow), 5 (purple), and 7 (blue) continually rise to the late peak. However, even in these muscles, this continual rise is less rapid during the period corresponding to the constant amplitude period of muscles 3 and 6. It is thus not difficult to imagine that these shapes are all members of a related family.
of responses, with the individual differences arising from shifts along a continuum of some muscle property. Muscle 2, which appears to have only a relatively brief contraction corresponding to the late peak of the contractions of the other muscles, initially seems to be an outlier. However, examining this muscle’s force trace showed that it was indeed contracting throughout the motor neuron burst, but before time 0.6 s had not developed sufficient force to switch the ASI transducer to the isotonic mode in which muscle shortening is allowed (see METHODS). As such, this muscle’s contraction is actually also similar to those of the other six.

It is important to stress that this similarity does not contradict the high intermuscle variability shown in Hooper et al. 2006b; with respect to the quantitative measures used in that work (e.g., contraction amplitude, rise and fall slope), the contractions in each panel in Fig. 3 would be highly variable. Nonetheless, the shape similarities of the contractions within each panel in Fig. 3, similar across-muscle contraction shape similarities in response to changing motor neuron input shown in Fig. 1 of Hooper et al. 2006b, and the spike number dependence shown in Hooper et al. 2006a, lead us to believe that extensor muscle contraction is a deterministic function of spike input and that the intermuscle variability arises from variations in muscle intrinsic properties. Our goal is therefore to relate the contractions to the spike patterns that create them.

One immediately obvious difference between the spike patterns and the contractions is that the spike patterns were highly irregular on a spike to spike level (note the jagged nature of the

FIG. 3. Contractions do not always temporally mimic the spike frequency profiles that induce them. A–E: data for 5 motor neuron bursts. All panels; bottom plots, vertical lines at bottoms of plots are motor nerve stimulation times; jagged thin line is instantaneous spike frequency; thick red line is a 5-point smoothing of spike frequency data. Top plots: smooth thin lines are the contractions that the burst in the bottom plot induced in the 7 extensor muscles used here. Note that, although the contractions in each panel show considerable amplitude variation, each panel’s contractions generally have quite similar shapes. A: contractions show a continuously decreasing slope but no or only very short periods of constant amplitude. B–D: although the bursts that induced them show long-lasting (hundreds of milliseconds), substantial (up to 50%) declines in spike frequency, the contractions do not, instead showing sustained periods of constant amplitude. E: motor neuron burst was 2 peaked with a long-lasting, substantial intermediary decrease in spike frequency, but contractions had either a sustained amplitude (muscles: 3, orange; 6, green) or a continuous amplitude increase (muscles: 1, black; 4, yellow; 5, purple; 7, blue) during the spike frequency decrease.
interspike frequency profiles, black lines in bottom plots), but the contractions were very smooth. This immediately suggested that the muscles are low-pass filters on the time scale of burst interspike intervals (5–10 ms), a conclusion also supported by earlier work showing that in the rising phase of the muscle contractions spike number codes for amplitude, which also requires that muscle relaxation time constant be longer than the typical interspike interval (Hooper et al. 2006a).

Another prominent characteristic of these bursts was that, after an initial period of high-frequency firing, they all show spike frequency declines. This is particularly obvious in the smoothed spike frequency profiles, and we have highlighted corresponding declines in amplitude, even though the declines in spike frequency were large (up to 50%) and occurred over hundreds of milliseconds. The contractions instead showed $J$ rises of continually decreasing slope that continued until the end of the burst (Fig. 3A), 2 rises that lasted for approximately one half the contraction duration, after which the contractions continued at a constant amplitude until the end of the burst (Fig. 3, B–D), or 3 more complex responses in which the spike frequency decline was either associated with a constant amplitude (muscle 3, orange; muscle 6, green) or increasing amplitude (muscle 1, black; muscle 4, yellow; muscle 5, purple; muscle 7, blue) contraction (Fig. 3E).

We found these mismatches surprising. The work noted above on spiking irregularity provided only a lower limit on how slow the muscle dynamics were. Furthermore, regardless of muscle time constant, we did not expect slow filtering to transform declining spike frequencies into constant amplitude contractions. We instead expected that $J$ if the time constants were very slow, the contractions would respond as though driven with a constant input at the mean spike frequency, in which case the contractions would continually rise with a decreasing slope, as these bursts are not long enough to reach the muscle’s steady state contraction amplitude (see Fig. 5A), or 2 that for shorter time constants, the declines would appear in the contractions, albeit with a delay.

However, from our prior stomatogastric muscle work (Morris and Hooper 1998) with models in which a resistor and capacitor are connected in parallel (RC low-pass filters), we knew that the dynamic activity (i.e., the activity when the system is far from its steady state) of such models can sometimes be nonintuitive. We therefore built such a model for the extensor muscles. To ensure that we used inputs to the model that, for model runs in which model time constants and input slopes and amplitudes were within certain ranges and the rising portion of the input ended at a higher value than the falling portion began, the model produced outputs with sustained constant amplitude periods. Figure 6B shows the response of the model to such input (black lines) with model time constants of 0.4/0.04 (red), 0.3/0.03 (orange), 0.2/0.02 (yellow), 0.15/0.015 (purple), 0.125/0.0125 (thick green), and 0.1/0.01 s (blue). A wide variety of outputs occur, ranging from ones that showed a continual increase throughout both phases of the input (red, orange, yellow) to ones that rose during the input’s rising phase and had sustained constant amplitude periods during its declining phase (purple, green) to ones that increased during the input’s rising phase and declined during its declining phase (blue).

**FIG. 4.** Mean linearized extensor muscle spike burst. Black dashed line shows division between the rising (blue) and falling (red) phases of the burst. Slanting blue and red lines show linearized spike burst profile and vertical lines show spike train that would give rise to this profile. Linearized spike burst profile lines were determined as follows. In this analysis, as in most of our other work on the extensor muscle (Hooper et al. 2006a,b), time 0 was defined to be when the rising phase of the muscle contraction reached an amplitude of 0.05 (see abscissa and right ordinate axes in Fig. 2). Cubic polynomial fits (Fig. 2) to each burst’s spike frequency plot were used to define a dividing time between the rising and falling phases of each burst, and linear fits were made to spike frequency data in each phase. Subject to certain significance criteria, these data were averaged to obtain the mean linearized spike frequency profile shown here.
In one sense, it is obvious from seeing these curve that contractions with periods of constant amplitude must be present in this system—if for some parameter sets model output continues to rise during the input’s declining phase and for other sets model output declines during it, there must be intermediate parameter values with sustained periods of constant amplitude. However, this logic provides no intuitive sense of why these sustained constant output periods arise. Considering the equation that governs model output during the declining period of the output, \( V = [V_0 + R(b - RCm)][1 - e^{-t/RC}] + Rmt \) (see Appendix), where \( V \) is model output, \( V_0 \) is model output at the time the input shifts from rising to falling, \( R \) and \( C \) are model resistance and capacitance, \( t/RC \) is the beginning of the input’s declining phase, and \( m \) and \( b \) are, respectively, the slope and intercept of the declining portion of the input, is similarly unhelpful. Although there certainly could be values of these parameters for which over substantial times \( [V_0 + R(b - RCm)][1 - e^{-t/RC}] + Rmt \) is constant, inspecting this equation does not provide much insight as to when or why this would be true.

An intuitive understanding of why these sustained constant amplitude periods occur can be achieved, however, by a graphical analysis and considering the steady-state values of the system for the various time constants used. For the red, orange, and yellow (although at the end of the input, yellow has almost reached steady state) contractions, even during the input’s declining portion, for all input values, the steady-state value of the RC curve to which the model is driving is always greater than the model’s contemporaneous output. Thus even though the input is declining, model output continues to increase. For the blue contraction, alternatively, very shortly after the input’s declining phase begins, the model’s amplitude is greater than the steady-state values the model would attain for the values contemporaneously present in the input. The model therefore moves toward these lower steady-state curves throughout the input’s declining portion, and thus model output continually declines during this period.

For the contractions with sustained constant amplitude durations, alternatively, during the initial parts of the declining input, the steady-state values the RC circuit would achieve at those input levels are greater than the model’s contemporaneous actual values, and thus model output rises. However, because the steady-state values are not far from the model’s actual values, the rate of rise is slow. As the input continues to decline, eventually the steady-state values corresponding to the input levels are less than the model’s actual values, and thus model output falls. However, again the difference between model actual value and the corresponding steady-state value is small, and so the rate of fall is small.

Figure 6C shows these processes for the green contraction in Fig. 6B. The black lines in Fig. 6C1 again show model input. The other solid colored lines show model output for sustained input at the values indicated in Fig. 6C2. For instance, the red line in Fig. 6C1 shows model output for a sustained input/s level of 172.5 (red line in Fig. 6C2). The dashed lines in Fig. 6C1 show the steady-state levels the various colored lines achieve. Exactly consistent with the explanation given above, for the red and orange input levels (levels occurring, respectively, at the beginning, time 0.15 s, red asterisk, and 0.04 s after the beginning, time 0.19 s, orange asterisk, of the declin-
ing phase of the input), the steady-state values are greater than the contemporaneous model values (green line), and thus at these times, model output continues to slowly rise. The steady-state values for the purple (achieved 0.08 s after the declining input began, at time 0.23 s, purple asterisk) and yellow (achieved 0.11 s after the declining input began, at time 0.26 s, yellow asterisk) input levels bracket the contemporaneous model values, and so during this time, the model switches from rising to falling. The steady-state value of the blue (achieved at the end of the declining input, 0.15 s after the declining input began, at time 0.3 s, blue asterisk) input level is below the contemporaneous model value, and so at this time model output is falling.

These results suggested that slow temporal filtering alone could possibly explain the mismatch between declining spike frequency and constant muscle contraction amplitude that we found odd in the contractions shown in Fig. 3, B–E. We therefore tested whether a more realistic RC based model could well reproduce the experimental data. The fundamental characteristic of the model is that each spike induces a twitch that first relaxes slowly and (as in Fig. 5A) switches to relaxing quickly if another spike does not occur rapidly enough. The values for the relaxation time constants had already been determined (Fig. 5, B and C). To make the model as realistic as possible axonal conductance delay, time to twitch maximum amplitude, and delay to rapid relaxation beginning were also measured. The first two times were measured in the isometric (force) domain (Fig. 7A) because, under the experimental conditions used here, single spikes do not result in muscle shortening. This work showed that twitch amplitude did not begin to strongly rise until 0.012 s after the stimulation, which was taken as the axonal conductance delay. Rise to peak amplitude took an additional 0.02 s.

In the model, each spike therefore induced, after a delay of 0.012 s, a constant amplitude input that lasted for 0.02 s and was then set to zero. At every time step, this input was added to the prior twitch amplitude, which would, without relaxation, result in each spike inducing a twitch whose amplitude linearly increased for 0.02 s and then remained constant. However, for all nonrest length amplitudes, at each time step, a decrease in amplitude proportional to the twitch amplitude at that time step also occurred. Therefore as twitch amplitude increased in the first 0.02 s, the amplitude increase per time step decreased because more relaxation occurred at each time step and, at times >0.02 s, when the spike induced input was 0, twitch amplitude exponentially decreased to rest length. The combined effect of these two processes was that each spike produced a model contraction with an initial nearly linear increase followed by an exponential decline.

The final parameter that needed to be included in the model was when, after a spike burst, the transition from slow to rapid relaxation began. Figure 7B1 shows an example using tonic nerve stimulation. The smooth black line is the muscle contraction, and the bottom trace shows the last three spikes of the stimulation. Time 0 was set to the last spike of the train. The muscle did not begin to rapidly relax until 0.045 s later (gray rectangle); 0.012 s of this delay is caused by nerve conductance delay, but the additional 0.033 s is presumably a muscle intrinsic delay before switching between the muscle’s slow and rapid relaxation states occurs. Figure 7B2 shows grouped data from the seven muscles used in these experiments. These data show that delay to rapid relaxation has a large variation inside
individual muscles. Attempts to correlate these variations with when in the experiment the contraction was triggered, contraction amplitude, or burst spike number or spike frequency were unsuccessful. The mean delay to rapid relaxation was 0.064 ± 0.014 s, or 0.052 s when the conductance delay is subtracted. This delay is large in comparison to extensor muscle bursts, with 52 ms comprising 13–17% of the 300- to 400-ms extensor motor neuron burst duration present in single middle leg walks.

A final complication arises from the fact that the Aurora force-length transducer does not allow shortening to begin until a preset muscle force has been reached. This means that the early part of the muscle contraction and the late part of its relaxation are not present in our movement recordings. We showed earlier (METHODS; Fig. 1) that exponential fits to the relaxation phases of the muscle contractions can be used to overcome this difficulty by defining an effective rest length that maximizes fit $R^2$ values. Models were run with these values as a rest length.

The model had two purposes. The first was to test whether slow filtering alone (using the 2 muscle time constants and various delays defined above) could reproduce the experimental contractions. To test this question, the five contractions of one of the muscles shown in Fig. 3 (muscle 6, green) were modeled. This muscle was chosen because its contractions generally (Fig. 3A being an exception) had mid-sized amplitudes and appeared to have approximately average shapes [the only other muscle that fit these criteria was muscle 1, black, but it is unclear it would be truly more representative, because it also has in 1 panel (Fig. 3C) an above-average contraction amplitude]. Muscle 6 also had an mean rise time constant close to the overall mean of all the muscles (Fig. 5B), although its mean relaxation time constant was considerably longer than the overall mean (Fig. 5C; also note that muscle 6's relaxations in Fig. 3 were generally slower than those of the other contractions). This difference, however, is not necessarily a drawback—if temporal filtering is capable of reproducing the contractions shown in Fig. 3, it must be able not only to reproduce the contractions of an “average” muscle, but of each individual muscle.

It is important in this work to make a distinction between two types of parameters present in model. One type is the temporal parameters—conductance delay, twitch rise duration, delay to rapid relaxation, and rise and relaxation time constants—that were experimentally determined. The last three of these parameters show wide variation in individual muscles. The hypothesis we are attempting to test is that slow filtering alone, using experimentally determined values, can reproduce the observed contractions, and we have been unable to find any correlations that explain the wide variations in these parameters. We therefore assumed that these parameters may be varying between the contractions shown in Fig. 3, and thus in the final modeling, we allowed these parameters to freely vary within 1 SD of the means shown in Figs. 5, B and C, and 7B2.

The other model parameter, which cannot be experimentally determined, is twitch amplitude. Given that our goal is to test how well temporal filtering alone can reproduce the data, it was important that twitch amplitude not vary between contractions (that is, allowing both twitch amplitude and temporal parameters to vary so as to maximize goodness of fit on a contraction by contraction basis would defeat our purpose). The question was how to determine what twitch value to use. The method we chose was to first run the models using the mean values of the delay to rapid relaxation and the rise and relaxation time constants and to find the twitch amplitude that with these values for the temporal parameters gave the best fits (see Fig. 8 for details of how goodness of fit was calculated). For muscle 6, this best twitch amplitude was 0.012 mm.

We then ran the models again allowing delay to rapid relaxation and rise and relaxation time constant to vary as specified above to achieve the best possible fit (Fig. 8). The fits appear to be excellent, and the mean of a normalized measure of the distance between the experimental and model data was only 8 ± 2%, with a maximum distance of 11% for Fig. 9E.
This exercise thus indicates that temporal filtering alone can well reproduce the muscle contractions for one muscle. Similar modeling of a random selection of the other muscle contractions show that the model can similarly well represent the other contractions (data not shown), which is not surprising given their shape similarities.

The second purpose of the model was to assess the possible functional significance of the wide range of delays to rapid relaxation and rise and relaxation time constants. That is, although these ranges appear wide (with CV—SD divided by the mean—in some cases as large as 0.5), this alone does not mean that these changes are sufficient to introduce large changes in contraction amplitude and shape. Model runs in which delay to rapid relaxation was varied showed that changing this parameter across the observed range primarily changed only when contractions began their rapid relaxation (note, for instance, that the beginnings of rapid relaxation of muscle 5, purple, in Fig. 3 were always delayed relative to these times in the other contractions). Although this variation was necessary to match model to experiment for some muscles, it is not otherwise interesting, and thus we do not show these data here. Variation of rise and relaxation time constant, alternatively, introduced substantial changes in both contraction amplitude and shape (Fig. 9).

In this figure the black lines are the contractions, the red lines are the best fits, the green lines are model runs with the rise and relaxation time constants set at the mean values of all the muscles plus (top green lines) or minus (bottom green lines) 1 SD, and the blue lines are model runs with the rise and relaxation time constants set at the longest (top blue lines) or

![Fig. 8](image-url) The model (red) well reproduced the contractions (black). Note that delay to rapid relaxation and rise and relaxation time constants were allowed to vary within 1 SD of overall muscle means (although, in general, values used were very close to the mean value; see Fig. 9). Goodness of fit was assessed by calculating the distance between contraction and model output at each time point (data were sampled at 0.0005 s and compared only for contraction amplitudes ±0.01). Distance between the two curves was calculated from

\[
d = \sqrt{\frac{1}{m} \sum_{i=1}^{m} (\exp^{i单单} - \mod^{i单单})^2}
\]

where \( i \) is time point number and \( m \) is total number of time points (for a detailed explanation of this technique, see Hooper et al. 2006a; Zhurov and Brezina 2006). Distances were normalized by dividing by each contraction’s mean amplitude.
shortest (bottom blue lines) values present in the data (see Fig. 5, B and C). It is important to make three points about these data. First, comparison of the red lines with the green in the five panels shows that, in all cases except the first, the best fit model value was close to the mean value, and thus obtaining the good fits in Fig. 8 did not require using unusual parts of the data range. Second, changing muscle time constant had large effects on model amplitude, which is not surprising given the major role interspike temporal summation plays in these contractions (see also Hooper et al. 2006a).

Third, changing model parameters within an SD of the overall mean value did not introduce large changes in contraction shape. That is, in Fig. 9A, the best fit and two green runs all resulted in contractions with a continuously decreasing slope without sustained periods of constant contraction amplitude. In Fig. 9, B–D, the best fit and green runs all had sustained durations of constant or near constant contraction amplitude (although the top green line in B is near to instead producing a contraction with continuously increasing amplitude, and the bottom green lines in C and D are either beginning or near to producing declining amplitude contractions). In Fig. 9E, the best fit and green runs all had an initial rapid rise followed by a period of decreased slope that was then followed by an increased slope to the contraction peak.

Changing model parameters across the entire range present in the data, alternatively, introduced clear, qualitative changes in contraction shape. For instance, the run with the fastest time constant in Fig. 9A (bottom blue line) resulted in a contraction with a sustained constant amplitude duration, something not present in other runs with this spike input. In Fig. 9, B–D, the runs with the slowest time constants resulted in continuously decreasing slope contractions without durations of constant amplitude (top blue lines) whereas the runs with the fastest time constants resulted in contractions with decreasing amplitude. In Fig. 9E, the run with the slowest contraction time constant is not qualitatively different from the best fit and green runs, but the period of intermediate slope at time ~0.1 s is clearly becoming less distinct. The run with the fastest contraction time constants, alternatively, is qualitatively different, now showing a decrease in amplitude at this time as opposed to an increase. These data thus show that, at least with respect to model output, the observed changes in muscle time constant are sufficient to introduce large changes in output amplitude and shape.
DISCUSSION

The goal of the work presented here was to determine if slow temporal filtering alone can explain the extensor muscle neuromuscular transform. The modeling work shows that, provided the different time constants governing muscle dynamics, twitch rise duration, and various delays in the system are properly accounted for, simple slow temporal filtering does appear to be sufficient. We also examined the effect of the wide variation in muscle time constants present in the data on model output. This showed that variations within 1 SD of the overall muscle mean values primarily affected only contraction shape, but variations across the entire data range introduced changes in both contraction shape and amplitude.

Comparison to prior work

A major conclusion of our earlier work on the extensor muscle was that there was no obvious canonical firing pattern of the early portions (those giving rise to the contraction rises) of extensor motor neuron bursts (Hooper et al. 2006a). The description of a mean extensor motor neuron burst here (Fig. 4) may thus be confusing to some readers. This difference primarily arises because of the different portions of the bursts that were analyzed and the different time scales over which the analyses were performed in the earlier work and here. The work in Hooper et al. 2006a dealt only with early portions of motor neuron bursts and used analysis techniques (return maps) that examine bursts on a spike pair by spike pair basis. The great spike to spike variability present in the bursts in Fig. 3, shown by the jagged appearance of the spike to spike (black lines) spike frequency profiles, is consistent with our earlier work showing no canonical small scale pattern in these bursts. That there is not strong long-scale patterning across larger portions of burst beginnings is also supported by 11% of the profiles not having an identifiable rising phase, and 51% of the linear fits to the rising phase portions not being statistically significant. The decreases in motor neuron spike frequency in the later portions of the bursts (portions not examined in the earlier work), alternatively, do appear to be generally present, as the linear fits to the declining portions were significant in 96% of the bursts.

However, even in this portion of the motor neuron bursts and when analyzed on this long time scale, great burst to burst variability was still present, as shown by the CVs of the various parameters defining the lines in Fig. 4, which ranged from 0.2 to 2.7, with an mean of 1. As such, although most bursts do show a declining spike frequency late in the burst, when this decline begins and ends, the frequencies from which it begins and at which it ends, and how rapidly it occurs, all show great variation. The mean linearized burst spike profile shown in Fig. 4 should thus not be interpreted as indicating most extensor motor neuron bursts lie near the blue and red lines. This figure is best interpreted instead as representing the possible physiological range of extensor motor neuron bursts, with any individual burst profile largely constrained to lie within the area defined by the plot axes, and most bursts having a declining phase at their end, but otherwise capable of assuming a very wide range of positions and slopes within it. Thus nothing in this study contradicts the conclusions of our prior work showing that extensor motor neuron bursts and extensor contractions show great step to step variability (Hooper et al. 2006a,b).

Presence of two time constants in extensor muscle contractions

Muscle contractions in response to spike trains result from the summation of the individual twitches each spike induces. If these twitches have a single relaxation time constant and summation is by simple addition, the summed contraction would have equal rise and relaxation time constants. One way in which two time constants could arise would be if the twitch relaxations had two time constants, as could arise, for instance, if calcium resequstration initially occurred slowly and then became rapid. The relaxation of extensor muscle twitches in the isometric domain, however, appear to be simple exponentials (average $R^2$ value of single exponential fits to the declining phases of the 7 large twitches in Fig. 7A; 0.95 ± 2), and last long enough that, were the time constant to change dramatically 0.6 s after the spike (the average delay to rapid relaxation), this change should be visible. Why extensor muscles have different rise and relaxation constants is thus unknown.

Relationship to muscle anatomy

Extensor muscles have singly (innervated by FETi), dually (innervated by SETi and Cl), and triply innervated fibers, with the singly innervated fibers primarily located in the proximal portion of the muscle, the dually in the middle, and the triply distally (Bässler et al. 1996). Histochemo staining identifies the singly innervated fibers as fast and the triply innervated as slow (Bässler et al. 1996). Muscle fiber length is 1.4 mm, with fibers from the middle of the muscle being shorter than those more proximal (Guschlbauer et al. 2007). As noted in METHODS, we believe that we are activating only or primarily the FETi fibers, but have seen little change in muscle response either isometrically or isotonically when we activate FETi and SETi or all three axons. Moreover, because muscle temporal properties and physiological contractions were always obtained from the same muscle and stimulation intensity was never changed after threshold was determined, any uncertainty on this score does not affect this article’s conclusions. One other issue raised by these observations is that one might expect this complex mixture of motor neuron and fiber types would require explicit modeling of the different fiber types and their (presumed) different dynamics. The extremely good fits shown in Fig. 8 suggest either that this is not true, or we were in fact activating only FETi in our stimulations.

Implications of wide variation in delays to rapid relaxation and rise and relaxation time constants

We were unable to find any correlation between these variations and stimulation order or paradigm or muscle state (e.g., contraction amplitude). One possible explanation is that these variations are truly random. However, the extensor muscle receives DUM neuron octopaminergic modulatory input, and many DUM neurons are rhythmically active during walking (Weller et al. 2005). The effect of octopamine on the extensor muscles is unknown, but it is possible that the observed differences arise from differing DUM input. The wide range of muscle dynamics we observe, in combination with this known modulatory input, thus raises the possibility that muscle dynamics are actively controlled in the animal to match...
muscle output to behavioral need, similar to the extremely well-studied situation in Aplysia feeding (Weiss et al. 1993).

Implications of linear changes in input in slow systems

Parallel RC circuits properties are typically presented by showing their response to constant driving inputs—an initial linear charging that then levels off—or to oscillating current—a slow filtering and phase shift of the input associated with a tonic amplitude increase if the current does not symmetrically oscillate around zero. Understanding how such circuits respond to other types of driving input was central to understanding the data presented here, and two aspects of this work should be further discussed. First, the input to the extensor muscles is, to a first approximation, a linear increase followed by a linear decrease, and it is thus important to describe how serial RC circuits respond to linearly changing driving inputs. The governing equation in a serial RC circuit driven by a current input is \[ I = \frac{V}{R} + C \frac{dV}{dt}. \] A linear input current is given by \[ I = m \times t + b, \] where \( m \) is the slope of the input and \( b \) its intercept. Substituting this equation into the first equation gives (with some algebraic manipulation) \[
\frac{m}{C} \times t - \frac{b}{C} = 0.
\] This equation can be solved (see APPENDIX) to give

\[
V = R \times (b - R \times m \cdot C) \times (1 - e^{-RC}) + R \times m \times t.
\]

This equation can be interpreted as follows. At early times, both the exponential and linear terms contribute to the voltage increase, and at later times, as the exponential term approaches zero, only a linear increase in \( V \) remains. Alternatively, it can be thought of as the classic RC charging curve, but the system is approaching not a steady-state value, but instead the line defined by \( V = R \times m \times t \). The activity of such a system can be somewhat seen in the blue trace in Fig. 6A, which (during the rising phase of the input) has a rapid initial increase that slightly levels off to a constant slope. This occurs because originally both the exponential and the linear term are contributing to model output, and later only the linear term does (for the other curves, the slope differences between the 2 regimes are too small to be seen, although the different slopes of the late linear portions are clear).

This analysis is interesting because it shows that linearly increasing spike frequency profiles will, within a few time constants of the muscle, result in linearly rising contractions (in the special case that \( b = R \times m \cdot C \), the rises will be linear at all times), as opposed to the contractions having the continually decreasing rise slopes that would occur with constant spike frequency inputs. Earlier work in which the rising phases of extensor muscle contractions were fit with linear functions showed that for many contractions such fits were very good (Hooper et al. 2006b). It is thus possible that the linear increase in extensor motor neuron activity seen in many steps occurs precisely to produce linearly rising contractions. The functional relevance of linear contraction rises is unclear, but these considerations suggest it would be worthwhile to investigate Carausius leg movements to determine whether their rise phases are also often linear.

The second important RC circuit–related issue is to stress the dynamic nature of the transformation of the declining inputs into constant amplitude outputs. That is, consideration of the equation governing model output (see RESULTS) shows when this transformation is occurring the terms \((b - RCm)(1 - e^{-RC})\) and \(mt\) must change approximately equally and oppositely as \(t\) changes. This requires that the exponential term has not decayed to near zero values, and thus can only occur during the early transient portion of system evolution far from the steady state condition. The slow time constant (RC) of the extensor muscle allows this transient portion of model activity to last for considerable time relative to total burst and contraction durations, and indeed none of the contractions in Fig. 3, B–D, ever reach steady state (at which they would decline with slope \( m \)), because their constant amplitude periods last until the end of the burst. Given that the rise durations of these contractions are less than the rise time constants of the muscles, the totality of these contractions is thus in the dynamic domain (defined as times in which the exponential term is not inconsequential). As noted frequently earlier, achieving an intuitive understanding of the activity of this (or any) system during such dynamic periods is not necessarily easy. However, for at least the extensor muscle, understanding this transient, non–steady-state component is essential to understanding muscle response to motor neuron input.

Implications for other systems

Many other invertebrate systems use graded, slow muscles [i.e., lobster swimmeret (Davis 1968), leech longitudinal (Mason and Kristan 1982), crustacean stomatogastric (Selverston et al. 1976), crab ventilation (Young 1975), Aplysia feeding (Weiss et al. 1992)]. There is no obvious reason that the data presented here should be idiosyncratic to stick insects, and thus many of these observations may have general relevance. In particular, these data suggest that it may be worthwhile to determine whether the muscles in these other systems also have different time constants during contraction and relaxation. These muscles may also produce large shape transformations of their input, and simple models based primarily on slow temporal filtering again may be able to reproduce well these changes (although in Aplysia a strong history dependence on twitch amplitude would also need to be included; Zhurov and Brezina 2006). Such systems may also work exclusively or primarily within the dynamic domain of their governing equations. Finally, it would also be interesting to examine motor neuron burst profiles in these systems to determine whether these profiles also have linear rises, which might suggest there is an as yet unappreciated functional advantage to linearly rising muscle contractions.

APPENDIX

How to determine the spike times that will give rise to a given spike frequency profile

Figure 4 shows the spike sequence that would give rise to the mean linear spike frequency profile presented there. Calculating the spike sequence that will give rise to a given spike frequency profile is not completely straightforward and we therefore provide here a general procedure for doing so. The key insight is to note that, by definition, a spike frequency function gives how many spikes occur per time unit (which is generally seconds but could also be, for instance, \( dt \)'s in the calculus sense) at every time point. It follows that the number of spikes that have occurred in any duration can be obtained by dividing the duration into a series of time intervals, multiplying each interval’s
mean spike frequency by the interval’s duration to obtain the number of spikes that occurred in the interval, and summing these spike numbers. In the limit, this procedure is equivalent to integrating the spike frequency function. [This procedure only works in this simple way if each spike pair’s frequency is defined to begin with the 1st spike of the pair and end with the last spike of the pair and is plotted at an abscissa value midway between the 2 spikes. If spike frequency is defined or plotted in other manners—as is, for instance, done in some data acquisition programs (e.g., Spike2) and was done in Hooper et al. 2006a—more complicated procedures must be used to derive spike times that will give a desired spike frequency profile. However, in the work presented here to which this analysis is relevant (Fig. 4), spike frequency was defined and plotted as above, and therefore this analysis is appropriate for our purposes.]

Our spike frequency functions are linear functions of time, spike frequency = mt + b, where t is time and m and b are the slope and intercept of the line. Integrating gives spike\(^n\) = ∫(mt + b)dt = \(\frac{mt^2}{2} + bt + C\), where C is a constant of integration. If spike 0 occurs at time 0, C is 0, and thus spike\(^n\) = \(\frac{mt^2}{2} + bt\), or 0 = \(\frac{mt^2}{2} + bt - \text{spike}^n\). We want to use this equation to solve for the times t at which 1, 2, 3, etc. spikes occur. Using the quadratic equation gives \(t = -b ± \sqrt{b^2 + 2m \cdot \text{spike}^n}\).

The sign of the ± must be positive because t = 0 when spike\(^n\) = 0, and so substituting the desired spike number into \(t = \frac{-b + \sqrt{b^2 + 2m \cdot \text{spike}^n}}{m}\) gives the spike’s time. Thus if b = 5 and m = -2, 3 spike occurs at \(t = \frac{-5 + \sqrt{5^2 + 2 \cdot (-2) \cdot 3}}{2} = 0.7\). In the work presented here, however, spike 0 occurred at the dividing time (div time) between the rising and falling phases of the spike frequency profile. Using this to solve for the constant of integration gives C = \(-\frac{m \cdot \text{div time}^2}{2} - b \cdot \text{div time}\) and thus spike\(^n\) = \(\frac{mt^2}{2} + bt - \frac{m \cdot \text{div time}^2}{2} - b \cdot \text{div time}\), or 0 = \(\frac{mt^2}{2} + bt + \frac{m \cdot \text{div time}^2}{2} - b \cdot \text{div time}\). The quadratic equation then gives \(t = \frac{m}{-b ± \sqrt{b^2 + 2m \cdot \text{spike}^n + b \cdot \text{div time}}\)}.

The sign of the ± is determined by noting that, from spike\(^n\) = \(\frac{mt^2}{2} + bt - \frac{m \cdot \text{div time}^2}{2} - b \cdot \text{div time}\) (see above), at t = 0, spike\(^n\) = -\(\frac{m \cdot \text{div time}^2}{2} - b \cdot \text{div time}\), and so the sign must be positive for the quadratic equation to be true at t = 0. The appropriate m and b values from the mean rise and fall spike frequency equations given in the Fig. 4 legend were substituted into this equation, and the times of negative (for the rising portion, which occurred before the dividing time) and positive (for the falling portion) spike numbers were calculated until sufficient spikes were obtained to match as closely as possible the rising and falling portion durations.

Provided spike frequency is defined and plotted as in the bracketed text in the first paragraph, analogous procedures can clearly be used to derive the spike times for any spike frequency function. For instance, the spike times for an exponentially rising function spike frequency = \(A \exp^{\alpha t}\) are given by spike\(^n\) = \(\int \exp^{\alpha t} \, dt = \frac{A \exp^{\alpha t} + C}{A\alpha}\). If spike\(^n\) = 0 at time = 0, then C = -\(\frac{A}{A\alpha}\) and so spike\(^n\) = \(\frac{A \exp^{\alpha t} - A\alpha t}{A\alpha}\).

Solving for t gives \(t = \tau \ln \left(\frac{\text{spike}^n + A\tau}{A\tau}\right)\); substitution gives the time of any desired spike.

Activity of an RC circuit driven by a linear current

Although solving the equation describing this situation is a standard technique, it may nonetheless be unfamiliar to many readers, and for their ease is included here. The master equation (see discussion) is \(\frac{dV}{dt} + \frac{V}{R} = 0\).

The first step in solving this equation is to find the “complimentary solution,” which is the solution of \(\frac{dV}{dt} + \frac{V}{R} = 0\). As can be verified by substitution, this solution is \(V_c = Xe^{-\frac{t}{RC}}\), where X is a constant that will be determined later. The next step is to find the “specific solution,” which in this case can be performed by assuming the solution has the form \(V_s = a_1 t + a_2\).

Substituting this into the original equation gives \(a_1 + \frac{a_1 t + a_2}{RC} = 0\). Because this equation must be true for all t, it follows that \(a_1 + \frac{a_1 t + a_2}{RC} = 0\) and thus \(a_1 = Rm, a_2 = RC(\frac{b}{C} - Rm)\), and \(V_s = Rm + R(\frac{b}{C} - RCm)\). The complete solution is the sum of the complimentary and specific solutions, or \(V = Xe^{-\frac{t}{RC}} + Rm + R(\frac{b}{C} - RCm)\), or \(R(\frac{b}{C} - RCm)\).

Similar substitution shows that, for the case in results in which the initial V, \(V_0\), was not equal to 0, \(X = V_0 + R(\frac{b}{C} - RCm)\), and \(V = [V_0 + R(\frac{b}{C} - RCm)\] \(1 - e^{-\frac{t}{RC}}\) + Rmt.

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