Sequences of predictive saccades are correlated over a span of approximately two seconds, and produce a fractal time series

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Abstract

We previously demonstrated that there is an abrupt (rather than smooth) transition between reactive and predictive modes of eye-movement tracking of target lights (a phase transition). We also found evidence that the sequence of eye movements in the reactive mode was independent, while those in the predictive mode were correlated and possibly formed a random fractal sequence. Here we confirm the finding of fractal structure by quantifying the rate of decay of nonlinear forecasting when applied to these data. We also estimate the window over which consecutive trials are correlated, and show that the duration of this window is fixed in time rather than number of trials. These results have implications for the neural mechanisms that drive predictive movements.

Keywords: Saccade, prediction, power-law scaling
1. Introduction

In previous work we demonstrated that there is a “phase transition” between two different modes of saccadic tracking (Shelhamer and Joiner 2003). Subjects were asked to follow a visual target that periodically jumped between two fixed positions at various rates. When target pacing was slow (below 0.5 Hz), saccade latency was approximately 180 msec; subjects waited for each target jump and reacted with a saccade. When target pacing was fast (above 0.6 Hz), saccade latency decreased dramatically, to the point where subjects made predictive saccades that anticipated the target jumps. We call these two distinct behaviors reactive and predictive saccades. The transition from reactive to predictive as pacing frequency increases (and vice versa) is not smooth but abrupt, and occurs at about 0.5 Hz. (This result is closely related to earlier work on phase transitions in finger tapping (Kelso et al. 1979).)

Our initial study also presented evidence for a difference in the statistical scaling properties of saccade latencies in the reactive and predictive modes. The power spectra of the series of latencies for reactive saccades was relatively flat, suggesting an uncorrelated or white noise process. The spectra of the predictive saccade latency series decayed as a function of frequency, in a power-law fashion. The decay exponent was in a range that suggested that the latencies form a random fractal sequence, indicative of long-term correlations between latencies. (See Discussion for an explanation of random fractal sequences.)

Here, we confirm and extend this result through the use of nonlinear forecasting or prediction (Farmer and Sidorowich 1987). Nonlinear forecasting is a way to predict subsequent events in a time series, based on a reconstruction of system trajectories in
state space. It has been shown that forecasting quality decays as a function of forecasting horizon (time into the future over which the forecast is made) in a characteristic manner for random fractal processes. We show that forecasting confirms that predictive saccade latencies indeed form a random fractal sequence, and we use this technique along with autocorrelation functions to determine the time interval over which past saccades have a “significant” effect on the latency of the current saccade. (A study with similar goals but using different methodologies was carried out for human finger tapping (Roberts et al. 2000).)

2. Methods

All subjects gave their consent to participate, as approved by the local Institutional Review Board. To obtain data on saccades at different pacing rates, subjects were seated in a dark room and presented with two red LED targets spaced horizontally 15 deg to the left and right of center. The LEDs were illuminated alternately at selected rates from 0.3 Hz to 1.2 Hz. The subject was instructed simply to look at each light when it came on. Eye movements and LED position were recorded, and a series of saccade latencies was extracted at each pacing rate by comparing the two time series. It is these series of saccade latency values, one series for each subject and tracking condition, that are analyzed throughout this study.

To obtain large data sets for the initial analysis of reactive and predictive behavior, 1000 trials were recorded from each of four subjects, during pacing at 0.3 and 0.9 Hz. To obtain data on the characteristics of predictive saccades at different frequencies, saccades from four subjects were recorded during pacing at 1.2, 1.0, 0.8, and 0.6 Hz, in
that order, for 150 trials at each frequency. The third subject in the first set of trials is the same as the fourth subject in second set; all other subjects are unique. The first set of trials here (N=1000 saccades) is identical to task 2 in our previous paper (Shelhamer and Joiner 2003), and the subjects are the same as in that study (although in a different order: subjects 1-4 here are subjects F, E, G, H from the previous paper).

Data are presented here in terms of both frequency of pacing and the interval between each target step (inter-stimulus interval, ISI, which is half the period). For frequencies 0.6, 0.8, 1.0, and 1.2 Hz, the corresponding ISI are 833, 625, 500, and 417 msec.

Nonlinear forecasting was carried out in a standard manner (Farmer and Sidorowich 1987) as used in our previous work on other aspects of eye-movement control (Trillenberg et al. 2001; Shelhamer and Gross 1998). Nonlinear forecasting is a means of predicting subsequent (latency) values, based on local approximations to a reconstructed system trajectory in state space. Forecasting begins by creating the system time trajectory in a suitable (possibly high-dimensional) state space. This is a space in which key variables are represented on each axis, and the system behavior over time traces out a trajectory in this space. Since it is not known what the key variables are, we use time-delay reconstruction (Packard et al. 1980; Ruelle 1990), which preserves essential properties of the actual state space but is more mathematically tractable. In this procedure, consecutively time-delayed values of a single measured quantity form points in an M-dimensional space. The reconstructed trajectory consists of the points $x(i)$, generated from the original time series $y(i)$ as follows:

$$x(1) = [y(1), y(2), \ldots, y(M)]$$
\[ x(2) = [y(2), y(3), ..., y(1 + M)] \]
\[ x(N) = [y(N), y(N+1), ..., y(N+M)]. \]

\( N \) is the number of points on the trajectory, \( M \) is the dimension. (More general forms of the reconstruction exist but this one is suitable for our purposes.) A reference point on the trajectory is chosen from which to make the forecast, and a set of \( K \) nearest neighbors is found. A linear regression is performed on these neighbors to see how they project into the future, and the resulting regression coefficients are used to forecast how the reference point will move in the future, over several steps. (A depiction of this procedure is described in Results in association with Fig. 2.) Since the subsequent points are in fact already known (but not used in the forecasting procedure), we can determine the correlation between the forecast values and the actual data, averaged over all forecasts made. The system can be characterized by the behavior of the correlation coefficient as the forecasting is carried further into the future. A random system will have uniformly poor forecasting quality for all forecasting steps, while a (non-chaotic) deterministic system will have uniformly good forecasting quality. Chaotic systems (those that are nonlinear and deterministic but unpredictable in the long run) and certain correlated random systems can be forecast in the short term but the quality decays rapidly.

Forecasting was carried out in this study from 1 to 20 steps (trials) ahead. For each data set, forecasting parameters were adjusted so as to optimize the forecasting quality while forecasting one step ahead. Optimization was performed via exhaustive search over a suitable range of values of state-space dimension \( M \), number of nearest
neighbors $K$, and the number of values preceding the reference point from which to choose these neighbors (the “template” from which to draw points).

A significant advance in the interpretation of forecasting results was made by Tsonis and Elsner (1992), who demonstrated mathematically that forecasting quality decays in a certain manner ($\log(1-r)$ vs. $\log($forecasting step$)$ is a linear function) if the underlying data form a so-called “random fractal sequence” or “fractional Brownian motion” (fBm). We apply this test to our forecasting results. The consequences of this particular data structure will be elucidated in the Discussion.

3. Results

*Large data sets – power spectrum, autocorrelation, forecasting*

Power spectra for reactive and predictive saccade latencies for one subject (N=1000 trials) are shown in Fig. 1A and B (based on Fig. 4 in (Shelhamer and Joiner 2003)). The reactive saccade spectrum (Fig. 1A) is approximately flat (slope $-0.00076$, approximating white noise), indicating that consecutive saccades during reactive tracking are uncorrelated. The predictive saccade spectrum (Fig. 1B), on the other hand, decays as a power law with respect to frequency (slope $-0.92$), producing a linear trend on the log-log scale. Power-law scaling indicates that consecutive saccades during predictive tracking are correlated in some manner.

Fig. 1C and D show the autocorrelation functions for the sequences of reactive and predictive latencies, corresponding to the power spectra in panels A and B. Autocorrelation shows how well a signal is similar to a time-shifted copy of itself. Here the abscissa is expressed in terms of number of trials rather than time *per se*. The
autocorrelation for reactive saccades (Fig. 1C) has a peak at zero, as expected, and
decays very quickly with inter-trial interval. This approximates a mathematical “(Dirac)
delta function,” and again indicates that trials are uncorrelated. (The autocorrelation for
uncorrelated or white noise is a single spike at zero lag.) The autocorrelation for
predictive saccades (Fig. 1D) likewise has a peak at zero, but decays much more
slowly, indicating that there is “significant” correlation (defined here as correlation > 0.2)
between saccade latencies even as far apart as 16 trials.

An example of forecasting for one subject tracking at 1.0 Hz is shown in Fig. 2. In
panel A, a small subset of 16 actual and 15 forecast latency values is presented, drawn
from the larger sequence. Actual and forecast data are shown, indicating the quality of
the short-term forecasts in this case; the actual and forecast data diverge after 10 steps.
In panel B, a two-dimensional reconstructed state-space trajectory is shown, for both
actual and forecast latency values. Here, only the first five latency values from panel A
are shown. This is a two-dimensional time-delay reconstruction: latency for a given trial
is plotted along the abscissa while latency of the previous trial is plotted along the
ordinate. As in panel A, the general trend of the forecast data (thin line, x) follows that of
the actual data (thick line, o). The dotted line marked by squares shows the template
data: latency data from early in the sequence which provide the basis for the forecasts.
To visualize the forecasting process, begin at the point labeled “1”, from where the
forecasting begins. The actual data trajectory moves down and to the left to get to point
“2”. The forecast trajectory, however, moves down and to the right. This is because the
forecast is based on a linear regression applied to the nearby template points – those
marked by squares along a trajectory that parallels the forecast trajectory from “1” to “2.”
In similar fashion, subsequent forecasts roughly parallel the nearest template trajectory, which generally also matches that of the actual data in this case, resulting in high-fidelity forecasts.

Forecasting quality from this same subject, and three others, is shown in Fig. 3. For reactive saccades (pacing at 0.3 Hz, N=1000), the correlation coefficients $r$ between sets of forecast and actual values are plotted as a function of the number of steps (trials) into the future over which the forecasting is carried out, using linear scales (Fig. 3A). Forecasting quality is very poor; even forecasting one step ahead is not possible with great fidelity. (Forecasting would be perfect for $r=1$ and completely nonexistent for $r=0$.) This again is representative of uncorrelated trials, since past behavior provides little information with which to forecast future behavior. A similar plot for predictive saccades (pacing at 0.9 Hz, N=1000) shows very good forecasting quality which decays gradually over the course of 10 to 15 trials (Fig. 3B). These same predictive-saccade forecasting results are plotted in Fig. 3C in the form of $\log(1-r)$ vs. forecasting step, and in Fig. 3D in the form of $\log(1-r)$ vs. $\log($forecasting step$)$. The decay appears as a linear function only in the case of Fig. 3D; this indicates that the data form a fractional Brownian process (Tsonis and Elsner 1992), the implications of which will be discussed below. Thus both forecasting and autocorrelation demonstrate that latencies of predictive saccades are correlated across at least five trials (during pacing at 0.9 Hz).

**Small data sets – autocorrelation and forecasting define a “correlation window”**

Inter-trial correlations among predictive saccade latencies were noted above. We now wish to determine if this “correlation window” is of constant duration (in seconds) during
predictive saccade tracking at different pacing frequencies, or if it is rather defined in terms of a constant number of trials regardless of the pacing frequency. In order to perform this analysis, we examined autocorrelation functions and forecasting quality, as above, over a range of predictive tracking frequencies from 0.6 to 1.2 Hz in four subjects. These data sets were obtained in a single recording session for each subject, and so were limited to N=150 trials (saccades) at each frequency in order to reduce fatigue. Thus the quality of the results, especially in terms of forecasting, is not as good as in the case of the larger data sets just presented. Nevertheless the appropriate trends are clear. (Although forecasting with such small data sets yields insufficient statistics to confirm power-law scaling per se, it does allow for comparisons across tracking at different frequencies.)

Predictive tracking was verified for each of these four subjects at each frequency by finding the mean latency in each case, after rejecting the first 10 saccades in order to analyze only steady-state behavior. Mean latencies (± s.d.) for the first subject are -57.41±52.1, -55.9±38.3, -37.7±33.8, and -38.1±38.7 msec, at 0.6, 0.8, 1.0, and 1.2 Hz respectively. Values for the other subjects are: 96.5±110.9, -17.6±153.0, -90.4±136.7, and -87.9±90.4 msec (subject 2), 25.6±111.7, -61.7±100.7, -60.2±77.5, and 78.8±49.4 msec (subject 3), and -75.4±87.9, -70.9±75.1, -71.0±56.3, and -7.3±48.6 msec (subject 4). Since latency is typically about 200 msec for random saccades (Becker 1989), a latency much smaller than this is deemed to be predictive. In particular, negative latencies correspond to responses that precede the stimulus (e.g., Fig. 7G). Even though latency is sometimes positive, all subjects exhibited predictive tracking at all frequencies.
Similarly to Fig. 1D, Fig. 4 shows the autocorrelation functions for sequences of 
N=140 predictive saccade latencies (again the first 10 were rejected in each case to 
eliminate the starting transient), for pacing frequencies 0.6, 0.8, 1.0, and 1.2 Hz, for four 
subjects (one along each row of graphs). The width of the central peak changes 
systematically with pacing frequency. The abscissa is expressed in terms of “trials” not 
“time,” and so the graphs depict correlations as a function of number of trials separation, 
not number of seconds separation. Taking the first points at which each autocorrelation 
magnitude decays to 0.2, we can define a “correlation window” as the corresponding 
width (actually the half-width, from zero lag to the 0.2 point) of the central peak at these 
points. These window sizes are provided in Table 1 for each subject at each pacing 
frequency, in terms of trials and in terms of the corresponding time in seconds (which is 
equal to the window width expressed in trials multiplied by the inter-stimulus interval, 
ISI).

These data are also plotted in Fig. 6A, where it is clear that the duration of the 
correlation window remains relatively constant in terms of time, not number of trials, 
across these different rates of predictive saccade pacing. For the first subject, the 
correlation window spans a range of 2.2:1 expressed in terms of trials (2.3 to 5.0, for a 
ratio of 5.0/2.3 = 2.2:1; in this notation a value of R closer to 1 indicates a smaller 
range), but only 1.6:1 when expressed in terms of absolute time (1.3 to 2.1 seconds). 
Similarly for the other subjects (except subject four where the values are close), the 
correlation window is approximately constant in terms of time (1.3:1, 1.3:1, 1.4:1) more 
so than trials (2.1:1, 2.3:1, 1.2:1). (Obvious outliers are excluded: 0.6 Hz for subject 
three and 1.2 Hz for subject four.) Each of these lines was tested for trend via linear
regression of the window size on frequency. The regression slopes were tested for a significant difference from zero (no trend across frequency) with t-tests at a significance level of 0.1. In all cases but one the slopes in terms of trials are significantly different from zero, and in all cases the slopes in terms of time are not significantly different from zero. This confirms that the correlation window is constant in terms of time more so than in terms of trials.

Forecasting quality for different pacing frequencies is shown in Fig. 5 for the same four subjects. There is significant quality to the forecasting in each case, indicating that predictive saccades were being made. Again using 0.2 as the threshold, correlation windows are provided in Table 2, and plotted in Fig. 6B. As in the case of the autocorrelation functions, the “correlation window” of good-quality forecasting varies with pacing frequency so as to remain approximately constant in terms of time more so than in terms of number of trials. As for the correlation window determined from the autocorrelation function, the lengths of the correlation windows determined from forecasting for each subject are approximately 1.5 to 2.5 sec.

Another way to picture the operation of this correlation window, over which past saccades influence the current saccade, is by examining the properties of saccades that outlast the stimulus. In Fig. 7 we present, for the same subjects and conditions as in Figs. 4 and 5, the final two seconds of target pacing in each case. Target pacing ends at the 2-sec point in each graph. It is very clear that the number of saccades that continue past the time of target presentation increases as the pacing rate increases. In accord with the results above, these continuing saccades last for about 1.5 sec regardless of pacing rate (the exception being panels E and I).
4. Discussion

Our earlier study (Shelhamer and Joiner 2003) indicated that sequences of predictive saccade latencies are correlated, and the present study confirms and extends that finding. During saccadic tracking of targets paced at high rates (typically above 0.5 Hz), target steps come often enough that there is insufficient time to react to each individual step when programming a saccade, and therefore the performance of previous saccades must be considered in triggering the current saccade. This leads to the correlations that we observe in the latency series of predictive saccades.

An obvious question is whether or not the repetitive rhythmic behavior addressed here has relevance for normal activities, which are often transient. Indeed these repetitive patterns may take the system out of its normal natural regime. However, we gain by stimulating the system in such a way that its behavior can be investigated with existing mathematical techniques. This is necessary at this stage of our investigations. Below we make some comments as to differences between long-duration and short-duration repetitive stimulation, which have some bearing on this question as well.

Saccade latency series as a fractional Brownian motion

The specific form of the correlations between predictive saccades results in what appears to be a random fractal sequence. Also known as “fractional Brownian motion” (fBm), this is a generalization of Brownian motion (Bassingthwaighte et al. 1994; Mandelbrot 1983; Addison 1997). Brownian motion is essentially the integral of Gaussian white noise. It has a power spectrum that decays as $1/f^2$ across all
frequencies. It is also statistically self-similar, in this sense: taking a small segment of a larger Brownian motion signal and expanding its time scale by (for example) a factor of $b=4$, the resulting signal will have the same statistics as the original if the amplitude scale is adjusted by a factor of $b^{0.5}=4^{0.5}=2$. This “expansion” exponent, 0.5, is called the Hurst exponent $H$, and is related to the decay exponent $\alpha$ of the power spectrum. This self-similarity means that the signal is a fractal.

Fractional Brownian motion is a generalization of this, where the Hurst scaling exponent $H$ can take on any value between 0 and 1. The associated power spectrum also decays with a value other than 2 (in the example in Fig. 1 the frequency-decay exponent is 0.92; that is, the spectrum has the form $1/f^{0.92}$). Values of $H$ other than 0.5 (regular Brownian motion) indicate that the signal exhibits long-term correlations (fBm). (For signals with long-term correlations such as fBm, the relationship $H=(1+\alpha)/2$ should hold (Rangarajan & Ding 2000). The first example given above is regular Brownian motion, which does not exhibit long-term correlations and for which this relationship does not hold).

In theory, Brownian motions (conventional and fractional) have infinite memory – the current value depends on integration over the infinite past. (Here we interpret the series of saccade latencies for each trial as a series of samples from an fBm.) Obviously this is not possible for real neural systems, and characterization of latency series as an fBm is necessarily an approximation. We set an arbitrary threshold in order to define a correlation window over which past saccades have “significant” impact on the current saccade. Since evidence that latency series are fBm suggests that long-term correlations exist, setting this strict threshold is subject to debate. It is more likely that
the performance of past saccades is taken into account in a decaying manner, such that more recent saccades have more impact. This evidence also implies that, as more trials are added to the history of the system (i.e., as predictive saccade tracking continues for many seconds or minutes), the effective correlations will extend further into the past and the correlation window will increase in duration. We return to this point below.

Another consequence of the fBm formulation (alluded to above) is that there are interactions between trials over different time scales. One might gain an appreciation of this from Fig. 8, where we have plotted the latency series (N=1000) from one subject during reactive tracking at 0.3 Hz (top) and predictive tracking at 0.9 Hz (bottom). While the top graph resembles an uncorrelated random sequence, the bottom graph exhibits features that are characteristic of fractional Brownian series, most notably fluctuations on different time scales. These observations suggest that there is distributed neural control of predictive saccades, operating over different time scales (Ding et al. 2002) and possibly coinciding with distributed neural saccade pathways (Gagnon et al. 2002; Gaymard et al. 1998).

**Extent of correlations between predictive saccades**

An obvious question is over what extent “significant” correlations exist. We examined the autocorrelation structure of predictive saccades at different pacing frequencies. The correlation window, as defined by autocorrelations greater than 0.2, was about 2 seconds. (In figure 1D for tracking at 0.9 Hz note that the correlation window is on the order of 16 trials or 8.9 sec, much longer than the windows found in the shorter data sets in Figs. 4 and 5. We conjecture that, as predictive tracking continues for long
periods, the correlation window may gradually increase as it becomes clear that it is safe to take into account performance further in the past since the stimulus conditions are likely to remain fixed. In the pursuit system, a similar neural store has been hypothesized to account for the ability to generate improved anticipatory pursuit after repeated viewing of target motion (Wells and Barnes 1998). The length of this store is apparently just a few seconds. As we noted above for saccades, the duration of this pursuit store can be greatly increased under suitable conditions (Chakraborti et al. 2002).

Another measure of correlation, which is sensitive to more general nonlinear interactions, is forecasting in the state space – how well can previous latency data be used to predict future latencies? Our findings here again show that correlations exist over about 2 seconds. The fact that the autocorrelation and nonlinear forecasting methods give similar results suggests that the dominant relations between trials are linear.

In this study we are faced with the typical conundrum of data set size. The quality of the autocorrelation functions, and especially of the forecasting results, would be improved by the use of larger data sets. However, our own data (autocorrelations in Fig. 1 versus Fig. 4) demonstrate that there can be large differences in the correlation structure of predictive latencies when prediction is carried out for more than a few seconds at a time. One might argue that the smaller data sets are more representative of natural behavior, where predictive repetitive motions might occur over several seconds rather than several minutes. Thus our results may characterize prediction in
natural circumstances more so than if we had used longer data sets with better
correlation and forecasting features.

Outline of a model for predictive saccade generation

There appears to be a relatively fixed “correlation window” over which past experience
is incorporated into the programming of the current saccade. This window, on the order
of 2 seconds, is fixed in absolute time rather than number of trials. We suggest that the
saccadic system attempts, whenever possible, to make predictive saccades based on
past performance. It does so by “integrating” – in some unknown manner – performance
over the previous few seconds. When the rate of target pacing falls below approximately
0.25 Hz, there are no longer any previous saccades within this correlation window at the
time that a target moves. Thus there is no information on past performance available for
adjustment of the current saccade, and prediction breaks down. (See Fig. 9 for a
schematic representation.) The system wants to predict, but the information available
for it to do so is too far in the past to be of value. An implication of this model is that
predictive saccade pathways are always in some sense “active,” though they have
nothing to process until target pacing reaches a critical value. This provides a natural
explanation for the abrupt “phase transition” that we previously demonstrated between
reactive and predictive saccades when the rate of target pacing reaches approximately
0.5 Hz (Shelhamer and Joiner 2003). This formulation might also explain the observed
preference for predictive over reactive behavior which may be a general feature of
repetitive tasks (Joiner and Shelhamer 2004).
There is an apparent discrepancy between our previously observed phase-transition frequency of ~0.5 Hz, and 0.25 Hz as predicted here by a correlation window of 2 sec. This might be resolved in several ways. First, as noted, the correlation window is arbitrarily defined by thresholds of 0.2; higher values would lead to shorter windows and a higher predicted transition frequency. Second, it may be that more than one previous saccade must fall within the correlation window in order for predictive tracking to occur. Establishing the details of this prediction model is a subject of our current work.

Acknowledgements

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References


Figure legends

Table 1. Length of correlation window based on analysis of autocorrelation functions shown in Figure 4.

Table 2. Length of correlation window based on analysis of forecasting results shown in Figure 5.

Figure 1. Power spectra for sequences of (A) reactive (0.3 Hz) and (B) predictive (0.9 Hz) saccades, from 1000 trials of each. Corresponding autocorrelation functions are shown in panels C and D.

Figure 2. Example of forecasting of one section of a latency time series. Panel A is a plot of sequential latency values from one subject tracking at 1.0 Hz, and corresponding forecast values using the first value as a starting reference point. Panel B is a two-dimensional time-delay reconstruction of the state-space trajectory followed by five saccades from the sequences shown in panel A. Abscissa is latency for a given saccade, ordinate is latency for the preceding saccade. In each plot, the thick line with x symbols represents actual latency values, and the thin line with o symbols represents forecast data. Dotted line marked by squares represents template values (actual latency data) used to form the forecasts. See text for details.
Figure 3. Forecasting of (A) reactive (0.3 Hz), and (B-D) predictive (0.9 Hz) saccades, from 1000 trials, for four subjects (each represented by a separate curve in each graph). Panel A indicates poor forecasting of reactive saccades. For predictive saccades, panel B shows forecasting quality (correlation coefficient $r$) as a function of forecasting step (saccade trial) into the future, with linear scales. Panels C and D show log(1-$r$) as a function of (forecasting step) and log(forecasting step), respectively.

Figure 4. Autocorrelation functions of the sequence of saccade latencies obtained at different frequencies from four subjects (one along each row of the figure). Autocorrelations are plotted from –20 to +20 trials of relative shift between each latency series and a copy of itself. ISI is inter-stimulus interval, the interval between target movements (half the period of the pacing rate).

Figure 5. Results of nonlinear forecasting of the same data sets as in Fig. 4. Forecasting quality is expressed as the average correlation coefficient ($r$) between forecast and actual latencies.

Figure 6. A) Length of correlation window as a function of stimulus pacing frequency, drawn from autocorrelation functions in Figure 4 and data in Table 1. B) Length of correlation window drawn from forecasting results in Figure 5 and data in Table 2. Thin lines are in terms of trials (saccades), thick lines are in terms of time (seconds).
Figure 7. Each graph shows the final two seconds of tracking for each subject at each pacing frequency, demonstrating the presence of saccades that outlast the pacing targets. Target pacing ends at the 2-sec point in each graph, where the target position signal disappears. Thick line is eye position.

Figure 8. Examples of a sequence of saccade latencies (N=1000 trials) for one subject, from reactive tracking at 0.3 Hz (top) and predictive tracking at 0.9 Hz (bottom).

Figure 9. Schematic representation of the generation of predictive saccades based on a “correlation window” over which past saccades influence the generation of the current saccade. The figure depicts target pacing (top row) at three frequencies, increasing from left to right. The second row indicates that a target jump has occurred and that a saccade is to be made at that point in time. The third row shows the hypothesized “correlation window,” of fixed time width. At low frequencies (left), no previous saccades are in this window at the time that a new saccade is to be generated (only the current saccade is in the window, as shown); therefore there is no basis on which to make predictive saccades based on past performance. At higher frequency (far right), target jumps and saccades come frequently enough that one or more fall within the correlation window at the time that a new saccade is to be made – at least two saccades (the current and a previous one) must be within the window for prediction to take place. These earlier saccades then form the basis for setting the timing of the new saccade.
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Table 2.
Figure 1.
Figure 2.
Figure 3.
Figure 4.
Figure 5.
Figure 6.
Figure 7.
Figure 8.
Figure 9.